The Fed *Explicitly* Speaks: Numerical Inflation Targeting and Smooth Diagnostic Expectations

Nayeon Kang*

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Abstract

This paper examines the impact of the Federal Reserve's communication on short-term inflation forecasts. Following the Federal Reserve's adoption of an explicit inflation target in 2012, SPF respondents' four-quarter-ahead inflation forecasts display two notable behavioral shifts: (1) increased confidence in their beliefs and (2) less overreactive forecasts to news, aligning more closely with rational expectations. A key factor driving these behavioral shifts is the reduction in uncertainty about trend inflation. To support this claim, I propose a parsimonious inflation expectations model with smooth diagnostic expectations. The model captures changes in both the first and second moments of individuals' predictive densities, providing an explanation for the decrease in short-term forecast disagreement. In line with this mechanism, incorporating the expectations formation framework into the New Keynesian model demonstrates that the Fed's target announcement contributes to the stabilization of realized inflation, mitigating agents' overreactive belief updating.

Keywords: Diagnostic Expectations, Inflation Targeting, Uncertainty, Monetary Policy Communication, Survey

JEL classifications: C53, E37, E58, E70, D83, D84

*Research Centre of the Bundesbank, nykang.econ@gmail.com.

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1 Introduction

> "Explicit inflation targeting is characterized by the announcement of an official target for the inflation rate and by an acknowledgment that low inflation is a priority for monetary policy." (Goodfriend, 2004)

> "Should the FOMC then take the next step and announce this number to the public? Some have argued that such an announcement would be unnecessary because the Fed's implicit inflation objective is already well understood by the market. I am skeptical.... To reassure those worried about possible loss of short-run flexibility, my proposal is that the FOMC announce its value for the OLIR (optimal long-run inflation rate) to the public."(Bernanke, 2004)

Echoing the collective wisdom of numerous economists, the Federal Reserve first publicly announced its long-term inflation target of 2% in 2012. Since then, much of the literature has focused on the anchoring effect of this communication. However, disclosing the long-term inflation target does more than merely anchor long-horizon expectations; it also affects short-term inflation forecasts. Specifically, forecasters experience lower subjective uncertainty, as reflected in the shrinking second moment of their predictive densities, and their point forecasts—representing the first moment of these densities—become less overreactive to new information.

This paper explains the shifts in both moments through a parsimonious inflation expectations model. A key driver behind these behavioral changes is the reduction in uncertainty in forecasters' information sets, stemming from the transparent communication of long-term inflation goals. By publicizing the 2% target, the Federal Reserve provides transparent information about trend inflation, thereby reducing conditional uncertainty regarding the trend component of inflation given the available information. This, in turn, influences short-term inflation forecasts, as agents incorporate both trend and cyclical components in their predictions, bringing their expectations more in line with rational expectations.

This research sheds light on subjective uncertainty, a critical but often overlooked aspect of expectations formation. Subjective uncertainty refers to an individual's perception of the level of unpredictability or lack of certainty when making forecasts. It reflects personal beliefs, perceptions, or incomplete information, rather than objective measurement. While previous research has primarily examined how far point forecasts (i.e., the first moment) of short-run inflation deviate from long-term goals, less attention has been paid to this uncertainty. An well-anchored stable point forecast does not necessarily imply low uncertainty around the forecast. For instance, a forecaster may predict inflation near 2% while also considering high risks of both deflation and inflation, indicating a lack of confidence in the forecast. When subjective uncertainty is high, economic agents are less confident in their beliefs, leading them to place greater weight on extreme possible future outcomes, even in response to small shocks and noisy signals.

Kumar et al. (2015) highlight the importance of such confidence by proposing five distinct definitions of anchored expectations to evaluate whether the inflation expectations of New Zealand firms' managers were well-anchored. These criteria include: (1) average beliefs closely aligned with the target, (2) limited dispersion of beliefs across agents, (3) *agents' confidence in their beliefs,* (4) minimal forecast revisions, particularly for variables with longer forecast horizons, and (5) limited co-movement between long-term and short-term expectations. The third criterion is particularly relevant to subjective uncertainty, which serves as a measure of forecasters' confidence in their point forecasts. Specifically, it reflects the forecasters' belief that inflation will stabilize within a specific range in the future. If this range is not sufficiently constrained—implying a forecaster lacks confidence in his own belief—then even small disturbances could lead to deviations from the anchored point forecast, resulting in de-anchoring. Despite the importance of this factor, it has received relatively little attention in the expectations formation literature. I address how explicit quantitative communication enhances individuals' confidence and reduces subjective uncertainty, thereby indirectly contributing to another dimension of anchoring.

This research makes three key contributions. First, unlike prior studies focused on aggregate-level long-term forecasts, this paper examines how the Federal Reserve's policy shift affects individual inflation forecasts, especially short-term expectations formation. While aggregate forecasts have garnered substantial attention, individual forecasts responding to monetary policy changes remain underexplored. This paper suggests that transparent communication, which reduces uncertainty in the information set, influences not only the conditional mean of an individual's subjective forecast distribution but also its conditional variance, thereby impacting both moments jointly. Consequently, this study documents how transparent and accountable policy disclosures by monetary authorities can alter individuals' forecasting behaviors.

Second, I propose a parsimonious model to explain three empirical findings observed in survey data, specifically for four-quarter-ahead inflation forecasts: 1) a reduction in overreaction to news since 2012, 2) increased confidence in beliefs, and 3) decreased disagreement among forecasters, arising from enhanced rationality. This model, in which forecasters share all parameters but differ only in receiving heterogeneous signals, successfully replicates forecast patterns observed in actual data, delivering clear and interpretable insights.

Finally, by incorporating the diagnostic expectations framework into a standard New Keynesian model, this exercise demonstrates how the Fed's explicit inflation target curbs overreaction in expectations formation and contributes to stabilizing realized inflation. The model bridges individual belief distortions with aggregate outcomes, highlighting the stabilizing role of transparent policy communication.

This study builds on three strands of research to explore how policy communications influence individual forecasts. Foremost, I develop an expectations formation model, rooted in diagnostic expectations (Bordalo, Gennaioli and Shleifer (2018); Bordalo et al., 2019; Bordalo et al., 2020) and smooth diagnostic expectations (Bianchi, Ilut and Saijo, 2024), to explain the overreaction of point forecasts to news and the shifts in subjective uncertainty. Diagnostic expectations (DE), which are based on Kahneman and Tversky (1972)'s representativeness heuristic, have been instrumental in advancing our understanding of individual expectations formation. When new information arrives, as measured with respect to a reference distribution based on past data, memory selectively recalls more vividly past events that are more associated with, or representative of, that current news. Extending this framework by incorporating changes in uncertainty surrounding current and past beliefs, Bianchi, Ilut and Saijo (2024) emphasize that new information not only updates the point estimate but also changes the conditional uncertainty surrounding the forecasted variable. In the standard DE model by Bordalo, Gennaioli and Shleifer (2018), the reference distribution is centered on the conditional mean of the true density at the past point when it was generated. However, its variance matches that of the true density based on current information. Alternatively, Bianchi, Ilut and Saijo (2024) condition their model on only reference information, thus the reference distribution captures the uncertainty from the time when expectations were first formed, rather than reflecting the current level of uncertainty. They call this approach "smooth diagnostic expectations". A key feature of Smooth Diagnostic Expectations (Smooth DE) is that as current uncertainty declines relative to past uncertainty, expectations distortion lessens. This aligns with the Federal Reserve's explicit messaging on long-term inflation goals, which has reduced both subjective uncertainty and expectations distortion. DE has been applied to financial markets (Adam and Nagel, 2023; Bordalo et al., 2021; Maxted, 2023) and a small open economy business cycle model (Na and Yoo, 2024). It has also been extended by L'Huillier, Singh and Yoo (2023), who incorporate the New Keynesian framework and demonstrate that the DE model outperforms the rational expectations model in a medium-scale DSGE setting. Bianchi, Ilut and Saijo (2023) integrate distant memory into their model, showing that the interaction between actions and DE repeatedly triggers boom-bust cycles in response to a single initial shock.

Secondly, this paper closely relates to public communication strategies. Eusepi and Preston (2010) and D'Acunto et al. (2020) demonstrate that communication is more effective in shaping expectations when it emphasizes policy goals and targets rather than the specific tools used to achieve those goals. This approach is particularly impactful for less sophisticated demographic groups. They conclude that target-based communication enhances policy effectiveness and helps build public trust in central banks, which is crucial for the success of their policies. Similarly, Coibion, Gorodnichenko and Kumar (2018) find that firm managers respond more strongly to information about the central bank's inflation target compared to other forms of information. Their experiments reveal that firms make the most significant adjustments to their forecasts when provided with information about the central bank's inflation target or recent inflation figures, indicating that firms place greater confidence in signals regarding these targets. Coibion, Gorodnichenko and Weber (2022) further demonstrate that households revise their inflation forecasts more significantly in response to FOMC statements and inflation targets delivered by the Fed compared to USA Today news articles. Despite similar information being conveyed, the stronger response to FOMC statements suggests that respondents may discount some information presented in newspapers. Experimental evidence supports the notion that households' and firms' information sets are significantly influenced by clear guidance from monetary authorities on policy directions. Given these findings, it is reasonable to assume that long-term inflation targets serve as strong signals to professionals, who tend to pay closer attention to the Federal Reserve's public speeches and data releases. Recent studies by Coibion et al. (2024) and Kostyshyna and Petersen (2024) demonstrate that heightened uncertainty negatively impacts household spending in experimental settings[1]. Distinctively, I focus on both the first and second moments of the predictive density using extensive survey data.

Finally, to measure individual forecaster's subjective uncertainty, I rely on Ganics, Rossi and Sekhposyan (2024). Direct measures of expectations, such as point forecasts, are typically gathered as fixed-horizon projections in the survey data. The Survey of Professional

[1] Coibion et al. (2024) reveal that high uncertainty about economic growth reduces household spending, and Kostyshyna and Petersen (2024) show that uncertainty surrounding inflation similarly has a negative effect on spending.

Forecasters (SPF) also conduct fixed-horizon point forecasts surveys. However, the SPF collects density forecasts in a "fixed-event" format, making it difficult to comprehensively observe and understand both the fixed-horizon point forecast and the uncertainty surrounding it. Density forecasts in the SPF are provided for fixed events, with panelists predicting inflation and output growth for the current and following calendar years, meaning the forecast horizon changes each quarter. Since I focus on four-quarter-ahead inflation forecasts, the fixed-event nature of the SPF density forecasts limits their direct applicability. Ganics, Rossi and Sekhposyan (2024) address this issue by proposing a method to reshape fixed-event uncertainty into fixed-horizon uncertainty. To accomplish this, they suggest combining current-year and next-year forecast densities through a convex combination. Using the probability integral transform (PIT) criterion, they estimate the weights required for this combination, resulting in a correctly calibrated predictive distribution. While Ganics, Rossi and Sekhposyan (2024) focus on aggregate-level uncertainty, I extend this methodology to measure individual-level uncertainty. Several researchers have explored expectations uncertainty. Binder (2017) and Krüger and Pavlova (2024) introduce a new measure of uncertainty in probabilistic survey on expectations at the individual response level. Abel et al. (2016) find no consistent relationship between forecast uncertainty and the dispersion of individual respondents' point forecasts using ECB-SPF data. Other studies, such as Grishchenko, Mouabbi and Renne (2019), use dynamic latent factor models to jointly estimate inflation uncertainty and point forecasts.

The remainder of this paper is structured as follows. Section 2 provides an overview of the survey data and inflation realizations, the key macroeconomic variable of interest. Section 3 presents empirical findings on how individual expectation behavior changed before and after 2012. Sections 4 and 5 lay the theoretical foundations of DE and Smooth DE, and discuss the structural framework of this research. Section 6 analyzes the estimation results and, through simulation, assesses how well the Smooth DE model which incorporates the Federal Reserve's long term inflation target announcement replicates the observed data. Section 7 demonstrates that the key findings of this paper are robust regardless of the estimated fundamental parameters. Section 8 presents the New Keynesian model with Smooth DE and analyzes the responses of key variables to structural shocks.

2 Data

This study investigates professional forecasts using the Survey of Professional Forecasters (SPF), which is conducted in the middle month of each quarter. For instance, in the first quarter, questionnaires are distributed to panelists by the end of January, and responses are collected between the second and third weeks of February. The Philadelphia Fed took over the administration of the survey from the ASA/NBER in the second quarter of 1990, making the 1990Q2 survey the first one administered by the Philadelphia Fed.

To ensure data consistency and reliability, I exclude the period prior to 1990Q2 due to evidence suggesting that the same identification numbers may have been assigned to different forecasters. For example, some individuals participated, then abruptly dropped out for several periods, and later re-entered, suggesting potential inconsistencies in the assignment of identifiers. Unfortunately, due to the lack of hard-copy historical records from the early surveys, the Philadelphia Fed could not investigate these cases further[2]. Given my focus on individual forecasters' expectations, I exclude these problematic periods from the analysis.

I use point forecasts to measure the conditional mean—the first moment—of the predictive distribution and density forecasts to capture the conditional variance—the second moment—of the predictive distribution. For point forecasts, the SPF questionnaire collects projections for both the quarterly and annual levels of the chain-weighted GDP price index (PGDP). Appendix A presents the exact question asked. Survey participants provide PGDP projections in levels, and I use their responses from the first column (PGDP1) and the fifth column (PGDP5) to construct each forecaster's four-quarter-ahead inflation forecast.

$$\pi^i_{t+4,t} = 100 \times \left(\frac{PGDP5^i_t}{PGDP1^i_t} - 1 \right). \tag{1}$$

Although the Federal Reserve's Statement on Longer-Run Goals and Monetary Policy Strategy specifies a 2% inflation target based on the Personal Consumption Expenditures (PCE) measure, I do not use the PCE measure for four-quarter-ahead inflation forecasts. This is primarily because the PCE inflation survey only began in 2007, which would significantly reduce the available data. Additionally, the SPF survey does not ask for distributional forecasts, limiting its usefulness in measuring forecast uncertainty.

[2]See "4. Forecasts of Individual Participants" in Survey of Professional Forecasters Documentation from the Philadelphia Fed.

The SPF compiles respondents' probabilistic assessments of changes in the GDP price index, asking them to provide a probability distribution for forecasted outcomes. Appendix B shows the exact question posed, and Section 3.2 details the construction of a density forecast for inflation over a four-quarter horizon.

Not only short-term inflation forecasts but also long-term inflation forecasts are essential to this analysis. The dispersion of individual forecast errors for long-term inflation provides key insights into the magnitude of heterogeneous signal noise regarding trend inflation. If economic agents form forecasts in a similar manner, yet their forecast errors vary, this dispersion may reflect differences in the signal noise they receive. The variance in long-run forecast errors is thus a valuable measure for estimating the magnitude of this noise in the Federal Reserve's communication. The analysis uses the 5-Year PCE Inflation Rate (PCE5YR) forecast responses from the SPF, which align with the Federal Reserve's PCE-based target.

In Section 3.3, I test the predictability of forecast errors to assess the extent to which the first moment—the conditional mean—of the subjective belief density is updated rationally. This analysis requires individual-level data on both forecast errors and forecast revisions. Forecast errors are defined as the difference between realized and forecasted inflation for the same period, where realized inflation is calculated using the GDP price index. To ensure alignment with forecasters' information sets, first-vintage data from the Philadelphia Fed's Real-Time Dataset for Macroeconomists is used for realized inflation. For instance, the inflation rate from 2000Q4 to 2001Q4 is calculated by dividing $PGDP_{2001Q4}$ value, published in the first (advance) release at 2002Q1, by the $PGDP_{2000Q4}$ value from the same release. As the SPF survey is conducted between the last week of the first month and the second week of the second month each quarter, forecasters likely incorporate this first release into their updated information set and adjust their forecasts accordingly [3].

3 Empirical Evidence

3.1 Statement on Longer-Run Goals and Monetary Policy Strategy

On January 24, 2012, the Federal Reserve released, for the first time, the "Statement of Longer-Run Goals and Monetary Policy Strategy". This statement, updated annually each

[3] The Bureau of Economic Analysis typically releases advance estimates of the current quarter in the last week of the first month of the next quarter.

January, conveys three primary pieces of information. First, it declares that a long-term inflation rate of 2% based on the Personal Consumption Expenditures (PCE) measure is most consistent with the Federal Reserve's statutory mandate. As mentioned in the statement, the Federal Reserve anticipates that this will not only reduce economic and financial uncertainty and enhance the effectiveness of monetary policy but also ensure that the public's longer-term inflation expectations become firmly anchored. The objective announced at the beginning of the year is consistently reaffirmed in subsequent Federal Open Market Committee (FOMC) statements.

The second piece of information pertains to the Federal Reserve's efforts to achieve the maximum level of employment. Unlike the clearly defined quantitative long-term inflation target, the Federal Reserve does not specify an employment rate target. This is because the maximum level of employment is determined not solely by monetary policy but also by nonmonetary factors that influence the structure and dynamics of the labor market. Accordingly, rather than announcing a specific numerical target, the Federal Reserve confirms that policy decisions would be based on assessments of the maximum level of employment, considering various indicators. Additionally, the statement provides the most recent projection of the longer-run normal rates of unemployment[4].

Lastly, the statement underscores the Committee's aim to mitigate deviations of inflation from its longer-term objective, while also addressing deviations of employment from its evaluations of the maximum sustainable level. These objectives are typically complementary; nevertheless, in cases where they may conflict, the Federal Reserve commits to a balanced approach in pursuing both goals. This "balanced approach" remains open to interpretation, as the statement does not define specific metrics or weights for each objective. Instead, it suggests that deviations in employment from the Committee's evaluations will be treated with equal consideration as inflation deviations from the long-term target, allowing for flexibility in response to prevailing economic conditions. Over time, this statement has undergone modifications. For instance, in 2016, the Committee introduced additional language as follow:

> The Committee would be concerned if inflation were running persistently above or below this objective. Communicating this **symmetric inflation goal** clearly to the public helps keep longer-term inflation expectations firmly anchored,

[4]Information about Committee participants' estimates of the longer-run normal rates of output growth and unemployment is published four times per year in the FOMC's Summary of Economic Projections. The most recent projections, such as the median estimate of FOMC participants for the longer-run normal rate of unemployment at 4.6 percent, were omitted from the amended statement released in August 2020.

> thereby fostering price stability and moderate long-term interest rates and enhancing the Committee's ability to promote maximum employment in the face of significant economic disturbances.

The 2016 statement introduced a symmetric inflation goal, suggesting that the Federal Reserve was equally concerned about inflation falling below or exceeding the target. A further notable amendment occurred in 2020, when, in an uncommon move, the statement was revised in August, mid-year. Among the many changes, the following language is particularly noteworthy:

> In order to anchor longer-term inflation expectations at this level, the Committee seeks to achieve inflation that **averages 2 percent over time**, and therefore judges that, following periods when inflation has been running persistently below 2 percent, appropriate monetary policy will likely aim to achieve inflation moderately above 2 percent for some time.

At this point, the Flexible Average Inflation Target (FAIT) was introduced. The Federal Reserve shifted its focus away from symmetric concerns about inflation moving either above or below the target and instead reflected a willingness to allow inflation to overshoot 2%, aiming to offset the persistent low inflation below 2% in the long run. This approach indicates the Federal Reserve's commitment to achieving an average of 2% inflation over time. In addition, the statement also emphasized that achieving the goals of price stability and maximum employment in a sustainable manner requires financial stability. It noted that policy decisions would also reflect a balance of risks, including risks to the financial system.

Despite these changes in tone, every statement issued from 2012 through the latest version in 2024 has consistently reaffirmed the 2% long-term inflation target. This reflects the Federal Reserve's clear and consistent signaling to the public, reinforcing the credibility of its commitment to price stability and anchoring inflation expectations. Moreover, while the core message remains unchanged, subtle modifications within these statements have provided the public with indirect yet smooth updates on current trend inflation. This nuanced communication allows the Federal Reserve to maintain flexibility in responding to evolving economic conditions without undermining the stability of long-term inflation expectations.

3.2 Subjective Uncertainty of Four-Quarter-Ahead Inflation Forecast

Since the announcement of the statement, numerous studies have examined whether the Federal Reserve's goal of firmly anchoring the public's longer-term inflation expectations has been achieved (Binder, Janson and Verbrugge, 2023, Bundick and Smith, 2023, Orphanides, 2019, Buono and Formai, 2018). This section aims to empirically demonstrate that the statement has also influenced density forecasts, drawing on the theoretical foundations proposed by Ganics, Rossi and Sekhposyan (2024).

In each survey, participants provide annual inflation density forecasts for both the current and the following years, as illustrated in Appendix B. The first step is to construct the cumulative distribution function (CDF) for a fixed-horizon density forecast, four quarters ahead ($h = 4$)[5]. This CDF, denoted as $F^{h,C}_{i,t,q}(\cdot)$, represents individual i's forecast for h-quarters-ahead of the quarter preceding time t. It is formulated as a convex combination of two separate CDFs: $F^{0}_{i,t,q}(\cdot)$, which represents individual i's density forecast for the *current year*, and $F^{1}_{i,t,q}(\cdot)$, corresponding to the density forecast for the *next year*. Before forming this convex combination I fit a normal distribution to the each of the individual CDFs $F^{0}_{i,t,q}(\cdot)$ and $F^{1}_{i,t,q}(\cdot)$. I borrow notations from Ganics, Rossi and Sekhposyan (2024).

$$F^{h,C}_{i,t,q}(\pi) \equiv \omega^{h}_{i,q} F^{0}_{i,t,q}(\pi) + (1-\omega^{h}_{i,q}) F^{1}_{i,t,q}(\pi), \text{ such that } 0 \le \omega^{h}_{i,q} \le 1, q \in \{1,2,3,4\}. \quad (2)$$

where $\omega^{h}_{i,q}$ denotes individual forecaster i's unknown weight in quarter q on the current calendar year forecast. Estimating $\{\omega^{h}_{i,q}\}^{4}_{q=1}$ follows the methodology outlined by Ganics (2018), which is based on the principle that a density forecast is probabilistically well-calibrated if and only if its corresponding probability integral transform (PIT) follows a uniform distribution. Therefore the weights are calculated by minimizing the distance between the PIT of the combined distribution and the uniform distribution. Notably, the PIT is evaluated at h-quarters ahead realized inflation.

$$PIT^{h}_{i,t,q} \equiv F^{h,C}_{i,t,q}(\pi^{h}_{t,q}) = \omega^{h}_{i,q} F^{0}_{i,t,q}(\pi^{h}_{t,q}) + (1-\omega^{h}_{i,q}) F^{1}_{i,t,q}(\pi^{h}_{t,q}) \quad (3)$$

To calculate vertical difference between the empirical distribution function of the PIT and the CDF of the uniform distribution at quantile $r \in [0,1]$, I define :

$$\Psi_{i,\mathscr{T}}(r, \omega^{h}_{i,q}) \equiv |\mathscr{T}|^{-1} \Sigma_{t\in\mathscr{T}} \mathbb{1}\left[PIT^{h}_{i,t,q} \le r\right] - r \quad (4)$$

[5] This examination looks at annual inflation rate from quarter $t-1$ to quarter $t+3$

where $\mathcal{T}$ is the index set of an appropriate sample of size $|\mathcal{T}|$ and $\mathbb{1}[\cdot]$ denotes the indicator function. Thus, $|\mathcal{T}|$ corresponds to the total number of years in which forecaster i participates. However, this approach faces a challenge due to the small sample size. If $\omega_{i,q}^{h}$ is estimated separately for $q = 1, 2, 3, 4$, the amount of available data decreases, leading to considerable estimation uncertainty. This issue is particularly exacerbated in this study, as it focuses on measuring weights at the individual level. To address the small sample challenge, Ganics, Rossi and Sekhposyan (2024) propose an alternative method. Instead of estimating weights separately, they suggest parameterizing the weights using flexible exponential Almon lag polynomials, as outlined by Andreou, Ghysels and Kourtellos (2010). The weights are specified as follows.

$$\omega_{i,q}^{h} \equiv exp(\theta_{i,1} q + \theta_{i,2} q^2), \; q \in \{1,2,3,4\}. \tag{5}$$

In addition to this, I adopt a rolling window estimation scheme by taking $\mathcal{T} = s - R + 1, s - R + 2, \cdots, s$ where $s = R, R+1, \cdots, T$ is the last observation of a rolling window of size R, and T is the last available density forecast observation in i's responses. In this analysis, the rolling window size is set to 20. The parameterization in the equation (5) ensures positive weights while pooling PIT across different quarters using an exponential polynomial.

The weights are collected in the vector $\omega_i^h \equiv (\omega_{i,1}^h, \omega_{i,2}^h, \omega_{i,3}^h, \omega_{i,4}^h)$ and using this formulation, I estimate weights through the minimization of the scaled quadratic distance,

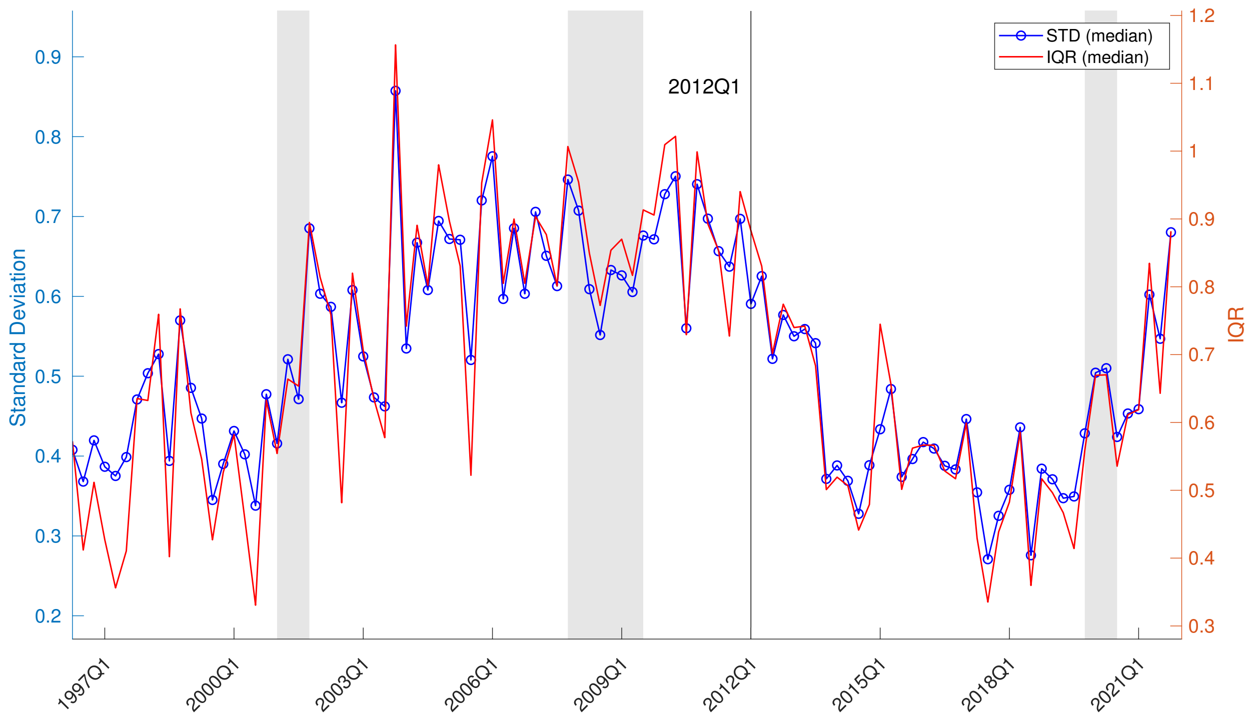
$$\hat{\omega}_{i,q}^{h} \equiv exp(\hat{\theta}_{i,1} q + \hat{\theta}_{i,2} q^2), \; q \in \{1,2,3,4\} \tag{6}$$

$$(\hat{\theta}_{i,1}, \hat{\theta}_{i,2})^\top \equiv \underset{\theta_{i,1},\theta_{i,2}\in\Theta}{argmin} \int_{\rho} \frac{\Psi^2_{i,\mathcal{T}}(r, \omega_{i,q}^h)}{r(1-r)} dr \tag{7}$$

where the parameter space Θ is chosen to ensure that the estimated weights satisfy $0 < \hat{\omega}_{i,q}^{h} \leq 1$ for $q \in \{1,2,3,4\}$, and they are non-increasing in q [6]. I use $\rho = [0,1]$ that is a finite union of neither empty nor singleton, closed intervals on the unit interval, in which domain I want to minimize the distance between the empirical CDF of the PIT and the uniform CDF.

Using the estimated weights $\hat{\omega}_i^h \equiv (\hat{\omega}_{i,1}^h, \hat{\omega}_{i,2}^h, \hat{\omega}_{i,3}^h, \hat{\omega}_{i,4}^h)$, derived from $\hat{\theta}_{i,1}$ and $\hat{\theta}_{i,2}$, the mixture distribution and its CDF $F_{i,t,q}^{h,C}(\pi)$ are obtained. The standard deviation of the fixed-horizon density forecast reflects subjective uncertainty. In Figure 1, the blue line

[6] The rationale behind this restriction is that, intuitively, as moving from quarter q to $q+1$, I aim to avoid assigning greater weight to the current year's forecast in $q+1$ than was assigned in quarter q



Note: The gray shaded areas indicate quarters in which there was an NBER-dated recession at any point in the quarter. Combined predictive densities for four-quarter-ahead inflation are derived at the individual level. The red line plots the median IQR for each period. The blue line with circles represents the median value of standard deviations for each period.

Figure 1: Subjective Uncertainty in Fixed-Horizon Forecast Densities

with circles shows the median of the individual standard deviations for each time period. These standard deviations are computed for each individual's density forecast, and the median is plotted. Notably, the standard deviation begins to decrease after 2012Q1 and remains low until economic volatility rises again with the onset of Covid-19.

To provide a more robust measure of subjective uncertainty, I also use the interquartile range (IQR) of each forecaster's density forecast. Unlike standard deviation, the IQR provides the central 50% of the distribution and is less affected by outliers, making it particularly useful for skewed or multimodal distributions. By looking into the IQR, I minimize the influence of irregularities, especially when two fixed-event distribution means differ significantly or outliers are present. The median IQR values offer a more robust measure of subjective uncertainty and are plotted as the solid red line.

Figure 1 illustrates that both the IQR and standard deviations confirm a sharp decline in inflation forecast uncertainty since 2012, suggesting that individual forecasters have become increasingly confident in their projections. It is important to note that this pattern does not result merely from data adjustments or transformations during the estimation process. To ensure a robust comparison, I calculate standard deviations from normal distribution-fitted fixed event densities for each survey vintage, avoiding the use of weighted averages. The median standard deviation is then derived for each quarter. The results reveal a significant reduction in subjective inflation forecast uncertainty across all horizons—from one to four quarters—since 2012, as further detailed in the Appendix C. This trend corroborates the findings in Figure 1.

This declining pattern cannot be simply attributed to stable economic environments. While some may argue that it results from the stabilized conditions following the Great Recession of 2007–2009, this is not necessarily the case. If economic stability were the sole driver, we would expect a similar reduction in forecast uncertainty across other key macroeconomic variables. To test this, I assess forecasters' subjective uncertainty in real GDP and civilian unemployment rate predictions around 2012. However, the probabilistic forecast survey for the civilian unemployment rate only began in the second quarter of 2009, limiting the available early data. As a result, the Ganics, Rossi and Sekhposyan (2024) methodology restricts the ability to observe changes in uncertainty before and after 2012 [7]. To address this limitation, I fit a normal distribution to fixed-event density forecasts for real GDP growth and the unemployment rate, deriving standard deviations and plotting the quarterly median. Appendix D shows that there are no significant differences in probabilistic forecasts for these variables around 2012. This supports the conclusion that the Statement on Longer-Run Goals and Monetary Policy Strategy, which clarifies long-term inflation targets, has a direct impact on inflation forecasts without affecting uncertainty for other macroeconomic variables.

3.3 Over-reaction of Inflation Point Forecasts

As the second moment of the predictive density decreases, it naturally raises the question of whether the first moment, or conditional mean (point forecast), is also affected, potentially causing shifts in forecast trends or directions before and after 2012. At the individual level, the average forecaster appears to overreact to private information, a phenomenon

[7] In the analysis, 20 quarters of survey data are used in a rolling window to estimate weights for density forecasts. For the civilian unemployment rate, responses up to 2014Q2 are used to produce the first fixed-horizon forecast for 2016Q1

Bordalo et al. (2020) empirically verify and explain through the DE model. Coibion and Gorodnichenko (2015) introduce the CG test to provide evidence of information rigidities, with the CG test coefficient reflecting the degree of rigidity, consistent with both sticky-information and noisy-information models. They demonstrate that, at the aggregate level, systematic predictability of forecast revisions on forecast errors results in a positive CG test coefficient when information rigidities are present.

In contrast, Bordalo et al. (2020) apply the CG test at the individual level and find a negative CG coefficient, indicating individuals' over-reactive expectations in response to news. In my analysis, following Bordalo et al. (2020), I apply the CG test at the individual level by dividing the data into pre-2012 and post-2012 periods, where a distinct declining trend of subjective uncertainty is evident in Figure 1.

The version of the CG test by Bordalo et al. (2020) is

$$\pi_{t+4} - \pi^i_{t+4|t} = \beta_0 + \beta_1(\pi^i_{t+4|t} - \pi^i_{t+4|t-1}) + \epsilon^i_{t,t+4} \tag{8}$$

where forecast revisions, $\pi^i_{t+4|t} - \pi^i_{t+4|t-1}$, quantifies new information received by individual i and $\pi_{t+4} - \pi^i_{t+4|t}$ represents individual i's forecast errors. If $\beta_1 > 0$, it suggests that the average forecaster underreacts to her own information, whereas $\beta_1 < 0$ indicates overreaction. A negative β_1 indicates that the average forecaster is excessively optimistic when forecast revisions are positive - that is, when the current news points to a more favorable future state compared to the previous information set. Importantly, under rational expectations, $\beta_1 = 0$, even in the presence of information frictions. In the individual-level CG test, β_1 does not directly indicate the presence or absence of information frictions. A rational forecaster may encounter information frictions stemming from inattention or noisy signals, but as long as the forecaster updates her beliefs rationally, forecast errors will remain unpredictable. If the forecaster has updated expectations rationally based on the available information, forecast revisions would not systematically predict forecast errors. Therefore, if there is no correlation between forecast revisions and forecast errors, it suggests that an average forecaster updates her expectations rationally, even with information frictions.

Table 1 exhibits that the pattern of overreaction in individual forecasts has weaken since 2012. Specifically, while β_1 turns positive after 2012, it remains statistically insignificant, suggesting that forecasters now form expectations closer to rational expectations for four-quarter-ahead inflation, with less sensitivity to new information. The Federal Reserve's additional communication on longer-run inflation has helped moderate overreac-

	1990Q2-2011Q4	2012Q1-2022Q1	1990Q2-2022Q1	1990Q2-2011Q4	2012Q1-2022Q1	1990Q2-2022Q1
β_0	-0.231^{***}	0.241	-0.062	-	-	-
	(0.079)	(0.238)	(0.105)			
β_1	-0.316^{***}	0.087	-0.147	-0.361^{***}	0.066	-0.194^{**}
	(0.070)	(0.175)	(0.103)	(0.069)	(0.128)	(0.090)
Obs.	2229	1142	3449	2221	1137	3438
FE	No	No	No	Yes	Yes	Yes

Note: CG test results using IV regression. Obs. indicates the sample size. Robust standard errors are in parentheses;***indicates significance at the 1% level. **indicates significance at the 5% level, and *indicates significance at the 10% level.

Table 1: CG Test Results at Individual Level

tion in short-horizon inflation forecasts. As a result, forecasters incorporate this new message, leading to a decrease in subjective uncertainty, which in turn boosts their confidence in their beliefs without exaggerating extreme forecast scenarios. In sum, the Statement of Longer-Run Goals and Monetary Policy Strategy has *jointly* influenced both the first and the second moments of the predictive density, particularly for short-term horizons.

This observation suggests that clearer and more consistent communication from policymakers has been key to moderating overreaction patterns typically seen in individual forecasting behavior. Even when compared to the real GDP growth rate and the unemployment rate CG test results provided in the Appendix E, this is a distinctive feature of inflation forecasts. The point forecasts for the real GDP growth rate and the unemployment rate tend to exhibit slightly stronger overreaction to news since 2012. To investigate the mechanisms driving these changes – observed uniquely in inflation forecasts – I present the analysis using the Smooth DE framework.

4 Diagnostic Expectations and Smooth DE

The key distinction between Smooth DE and DE lies in changes in conditional uncertainty. In Smooth DE, the degree of overreaction depends on the current level of uncertainty about the state relative to the reference uncertainty formed in the past. Before delving into the specifics of Smooth DE, it is important to first understand the foundation laid by the DE model, which serves as the basis for these extensions.

4.1 Diagnostic Expectations

Bordalo, Gennaioli and Shleifer (2018) introduce the DE framework, which explains how survey forecasts become overly optimistic following good news and overly pessimistic after bad news. This overreaction, especially prominent in credit markets, challenges the assumptions of rational expectations theory. As an alternative, the authors propose the DE model, which draws on Kahneman and Tversky (1972)'s concept of 'representativeness heuristic'. The DE framework integrates both overextrapolation and the neglect of risk.

In the DE model, forecasters reassess the likelihood of future outcomes based on 'representativeness'. When forming forecasts about future economic states, individuals operating under the DE mechanism do not assess the distribution of a future state using the true conditional distribution given current news or realizations. Although this information is stored in their memory, when new information arrives, they compare the likelihood of certain future states given current news (updated information set) to that derived from reference information which has not incorporated the news. Because of memory limitations, agents cannot recall information perfectly; instead, they quickly recall certain 'representative' states—specifically, those that seem more likely based on new information. These states are the ones whose likelihood increases the most when compared to their previous beliefs or 'reference memory', which was shaped by past information. As a result, individuals overweigh these representative states, distorting the objective likelihood. Bordalo, Gennaioli and Shleifer (2018) formalize 'representativeness' as

$$rep_t = \frac{h(\hat{\omega}_{t+1}|G)}{h(\hat{\omega}_{t+1}|-G)},$$

where $\hat{\omega}_{t+1}$ is the forecasted variable, G represents updated information, serving as a posterior group that incorporates the latest news, while $-G$ denotes reference information, which serves as a reference group without incorporating the latest news. A certain expected outcome $\hat{\omega}_{t+1}$ is more representative if it occurs more frequently given news (G) *relative to* the reference memory ($-G$), and this state $\hat{\omega}_{t+1}$ comes to minds faster than other possible states. This representativeness distorts the objective density in the minds of decision-makers, leading them to form a biased subjective density. The distorted subjective density is expressed as

$$h_t^{\theta}(\hat{\omega}_{t+1}) = h(\hat{\omega}_{t+1}|G)\left[\frac{h(\hat{\omega}_{t+1}|G)}{h(\hat{\omega}_{t+1}|-G)}\right]^{\theta}\frac{1}{Z}$$

where Z is a normalizing constant. As θ increases, the tendency to oversample representative states becomes stronger, resulting in greater distortion of the objective density. Building on this distorted density, the diagnostic belief is formalized as follows.

Proposition 1. *When the process for ω_t is AR(1) with normal $(0, \sigma^2)$ shocks, the diagnostic distribution $h_t^{\theta}(\hat{\omega}_{t+1})$ is also normal, with variance σ^2 and mean*

$$\mathbb{E}_t^{\theta}(\omega_{t+1}) = \mathbb{E}_t(\omega_{t+1}) + \theta[\mathbb{E}_t(\omega_{t+1}) - \mathbb{E}_{t-1}(\omega_{t+1})].$$

Proof. See Appendix in Bordalo, Gennaioli and Shleifer (2018). □

$\mathbb{E}_t^{\theta}$ represents diagnostic expectations, while $\mathbb{E}_t$, the expectation operator without the superscript θ, represents rational expectations. Both $\mathbb{E}_t(\omega_{t+1})$ and $\mathbb{E}_{t-1}(\omega_{t+1})$ represent the conditional mean of rational expectations from the true density. It is assumed that the variance of diagnostic distribution σ^2 is identical to that of the fundamental shocks. Under rational expectations, θ equals zero, and the DE model collapses to rational expectations. This implies that agents have no memory limitations, allowing them to recall information perfectly and update their beliefs rationally. On the other hand, when $\theta > 0$, diagnostic expectations overreact to the information. A positive θ means that agents evaluate the likelihood ratio based on representativeness, and θ measures the severity of this distortion. Due to the distorted probability density in their incomplete memory, agents' oversampling of representative states significantly influences their expectations.

Consequently, while individuals may hold rational expectations in the back of their minds, diagnostic expectations are unconsciously distorted by the representativeness heuristic. This heuristic causes forecasters to overemphasize certain aspects of the information received, thereby distorting the objective distribution in their forecasts.

4.2 Smooth Diagnostic Expectations

Bordalo et al. (2020) and Bordalo, Gennaioli and Shleifer (2018) assume that subjective uncertainty is equivalent to objective uncertainty. A key innovation in Bianchi, Ilut and Saijo

(2024)'s Smooth DE framework is the disconnection of objective and subjective uncertainty. They relax the rigid assumption that subjective uncertainty must mirror objective uncertainty. This shift acknowledges that if the first moment of expectations is distorted, it is reasonable to expect the second moment to be affected as well. Despite this intuition, the standard DE models do not focus on the role of uncertainty until Bianchi, Ilut and Saijo (2024) highlight the importance of changes in conditional uncertainty in shaping expectations.

The change in conditional uncertainty is represented as

$$R_{t+h|t,t-J} \equiv \frac{\sigma^2_{t+h|t}}{\sigma^2_{t+h|t-J}} \tag{9}$$

where $\sigma^2_{t+h|t-J}$ is the variance of the true density conditional on reference information set (in my model, reference information set is the information set from the immediately preceding period, $J = 1$) and $\sigma^2_{t+h|t}$ is the variance conditional on the current updated information set[8]. Forecasters retrieve memory selectively, leading to a distorted density $f^{\theta}(x_{t+h}|\mathscr{I}_t)$ affected by representativeness.

$$f^{\theta}(\hat{x}_{t+h}|\mathscr{I}_t) = f(\hat{x}_{t+h}|\mathscr{I}_t)\left[\frac{f(\hat{x}_{t+h}|\mathscr{I}_t)}{f(\hat{x}_{t+h}|\mathscr{I}^{ref}_t)}\right]^{\theta}\frac{1}{Z} \tag{10}$$

In my model $\mathscr{I}^{ref}_t$ is the information set updated in the preceding period, $\mathscr{I}_{t-1}$.

Proposition 2. *(Smooth DE) Consider the reference group given by density in equation $f(\hat{x}_{t+h}|\mathscr{I}^{ref}_{t-J}) = \mathbb{N}(\hat{x}_{t+h};\mu_{t+h|t-J},\sigma^2_{t+h|t-J})$. Denote the ratio variances for the current and reference groups as*

$$R_{t+h|t,t-J} \equiv \sigma^2_{t+h|t}/\sigma^2_{t+h|t-J}$$

If $R_{t+h|t,t-J} < (1+\theta)/\theta$, the Smooth DE density $f^{\theta}(\hat{x}_{t+h}|\mathscr{I}_t)$ in equation (10) is Normal with conditional mean

$$\mathbb{E}^{\theta}_t(x_{t+h}) = \mu_{t+h|t} + \theta\frac{R_{t+h|t,t-J}}{1+\theta(1-R_{t+h|t,t-J})}[\mu_{t+h|t} - \mu_{t+h|t-J}] \tag{11}$$

and conditional variance is

[8] $J \geq 1$ allows distant memory for reference information set.

$$\mathbb{V}_t^\theta(x_{t+h}) = \frac{\sigma^2_{t+h|t}}{1+\theta(1-R_{t+h|t,t-J})}. \tag{12}$$

Proof. See Appendix in Bianchi, Ilut and Saijo (2024). □

$\mu_{t+h|t}$ and $\mu_{t+h|t-J}$ represent the conditional mean of rational expectations from the true density. The term $R_{t+h|t,t-J}$ plays a critical role in both the conditional mean, $\mathbb{E}_t^\theta(x_{t+h})$, and variance, $\mathbb{V}_t^\theta(x_{t+h})$. In Bianchi, Ilut and Saijo (2024), they highlight three key features of Smooth DE as introducing the effective distortion parameter

$$\tilde{\theta}_{t,t-J} \equiv \theta \frac{R_{t+h|t,t-J}}{1+\theta(1-R_{t+h|t,t-J})}. \tag{13}$$

The effective distortion parameter $\tilde{\theta}_{t,t-J}$ measures how much the conditional mean, in effect, overreacts to new information. This time-varying parameter reflects how much uncertainty is resolved as new information is incorporated. When current information significantly reduces uncertainty compared to reference information formed in the past, the role of retrieved memory diminishes. As uncertainty decreases, reliance on representativeness is reduced, allowing forecasters to depend more on precise information about the current state. This results in a conditional density, $f^\theta(\hat{x}_{t+h}|\mathscr{I}_t)$, that is closer to the true density.

Within the standard DE framework, it is impossible to demonstrate that the distortion parameter varies over time; it remains constant throughout. In contrast, in the smooth DE model, the effective distortion parameter, $\tilde{\theta}_{t,t-J}$, evolves over time, influenced by changes in the level of conditional uncertainty. Furthermore, the standard DE model does not support the evidence that agents tend to exhibit lower subjective uncertainty as they receive more transparent signals. In the standard DE model, agents' subjective uncertainty always aligns with true uncertainty. The smooth DE model is crucial because it explains joint changes in the conditional mean and variance of the belief distribution by incorporating $R_{t+h|t,t-J}$.

5 Model

In this section, with smooth DE, I propose a parsimonious model of individuals' expectations formation for four-quarter-ahead inflation. It is assumed that agents update their beliefs about unobservable components upon receiving signals that convey both information about the underlying states and noise.

5.1 Inflation Dynamics

Inflation is modeled as the sum of two unobserved components: a permanent trend component τ_t and transitory cyclical component (i.e., the inflation gap) ε_t, which follows an AR(1) process with persistence ρ_ε.

$$\pi_t = \tau_t + \varepsilon_t \tag{14}$$

This trend-cyclical decomposition builds upon the foundational work of Stock and Watson (2007) and is further developed by more recent studies, including those by Chan, Clark and Koop (2018), Mertens (2016), Mertens and Nason (2020), Nason and Smith (2021). Mertens and Nason (2020) and Nason and Smith (2021) analyze inflation forecasts within a sticky information framework incorporating average forecasts, demonstrating that gradual adjustments in forecasts during the high-inflation period of the 1970s led to persistent forecast errors until the Volcker disinflation. Their work also highlights the increased stickiness in inflation forecasts following this period.

This paper contributes to the existing literature by applying a noisy information model, analyzing individual-level panelist forecasts instead of aggregate forecast data, offering a novel perspective on the individual level expectations formation.

5.2 State-Space Model

Forecasters make multi-period-ahead inflation forecasts by combining their predictions of the trend component, τ_{t+h}, and the cyclical component, ε_{t+h}. To generate these forecasts separately, agents update their beliefs about the current states $\tau^{i,\theta}_{t|t}$ and $\varepsilon^{i,\theta}_{t|t}$, based on the information available at time t. Forecasters update in a forward-looking way in the sense that forecasts take the variable's true persistence into account, even if they overreact to news[9].

[9] Note that, since the expectations formation rule is forward-looking, $\tau^i_{t+h|t} = 1^h \tau^i_{t|t}$, given the random walk process, and $\varepsilon^i_{t+h|t} = \rho^h_\varepsilon \varepsilon^i_{t|t}$, given the AR(1) process.

$$
\begin{aligned}
\mathbb{E}_t^{i,\theta}(\pi_{t+h}) &= \mathbb{E}_t^{i,\theta}(\tau_{t+h}) + \mathbb{E}_t^{i,\theta}(\varepsilon_{t+h}) \\
&= \mathbb{E}_t^{i,\theta}(\tau_t) + \rho_\varepsilon^h \mathbb{E}_t^{i,\theta}(\varepsilon_t) \\
&= \begin{pmatrix} 1 & \rho_\varepsilon^h \end{pmatrix} \begin{pmatrix} \tau_{t|t}^{i,\theta} \\ \varepsilon_{t|t}^{i,\theta} \end{pmatrix}
\end{aligned}
$$

where $h = 4$, $\tau_{t|t}^{i,\theta} = \mathbb{E}_t^{i,\theta}(\tau_t)$ and $\varepsilon_{t|t}^{i,\theta} = \mathbb{E}_t^{i,\theta}(\varepsilon_t)$ represent individual i's updated trend and cyclical components, respectively, both distorted by θ, given the information available at time t . The expectation operator $\mathbb{E}_t^{i,\theta}$ reflects individual i's Smooth DE. The parameter θ captures the extent of the departure from rational expectations.

The transition equation, a key part of the state-space model, remains unchanged before and after 2012, reflecting the (conservative) assumption that the data generating process for π_t does not change.

$$
\begin{pmatrix} \tau_t \\ \varepsilon_t \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & \rho_\varepsilon \end{pmatrix} \begin{pmatrix} \tau_{t-1} \\ \varepsilon_{t-1} \end{pmatrix} + \begin{pmatrix} \sqrt{1-\gamma}\sigma & 0 \\ 0 & \sqrt{\gamma}\sigma \end{pmatrix} \begin{pmatrix} u_{t,\tau} \\ u_{t,\varepsilon} \end{pmatrix} \tag{15}
$$

The total variance of the innovations to π_t, conditional on time $t-1$ information, is σ^2. The share of this variance attributed to shocks to the trend component τ_t is $1-\gamma$ while the remaining share γ is attributable to the cyclical component ε_t. $u_{t,\tau}$ and $u_{t,\varepsilon}$ are independent and follows standard normal distributions, $u_{t,\tau} \sim \mathbb{N}(0,1)$ and $u_{t,\varepsilon} \sim \mathbb{N}(0,1)$.

At each time t, the target variables $\tau_{t+h|t}$ and $\varepsilon_{t+h|t}$ are forecasted. To make these forecasts, forecasters must update their beliefs about the current states τ_t and ε_t, which are unobservable. Instead of direct observation, they rely on noisy signals that contain information about these states. Forecasters, therefore, infer τ_t and ε_t based on these signals. From this point forward, we assume the signal structure is exogenous.

5.2.1 Signal Structure Prior to the Statement: 1990Q2–2011Q4

Before 2012, agents receive only one signal that contains information about both τ_t and ε_t, but they cannot disentangle which portion corresponds to each component. Agents receive private signals, leading to heterogeneity in forecasts. Each agent's signal noise is drawn from a standard normal distribution $\nu_{t,\tau\varepsilon}^i \sim \mathbb{N}(0,1)$, and size of the noise is represented by $\sigma_{\nu,\tau\varepsilon}$

$$S^i_{t,\tau\varepsilon} = \begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} \tau_t \\ \varepsilon_t \end{pmatrix} + \sigma_{\nu,\tau\varepsilon} \nu^i_{t,\tau\varepsilon}. \tag{16}$$

I will henceforth refer to the signal $S^i_{t,\tau\varepsilon}$ as the 'mixed signal'.

5.2.2 Signal Structure After the Statement: 2012Q1–2021Q4

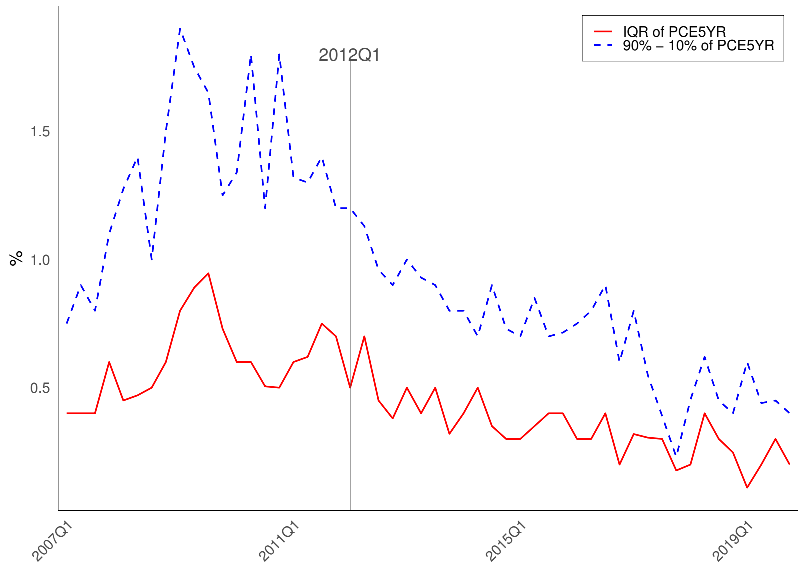
From 2012 onward, agents begin receiving an additional signal from the Statement on Longer-Run Goals and Monetary Policy Strategy, which clarifies the Federal Reserve's long-term inflation target. As its nuances evolve and shift over time, this signal indirectly conveys information about the current trend inflation, τ_t, while also providing the Federal Reserve's viewpoint on the current economic situation. The noise associated with this signal, $\nu^i_{t,\tau}$, varies across agents, reflecting different levels of trust in the Federal Reserve. For example, an agent with high confidence in the Fed's ability to maintain price stability would have $\nu^i_{t,\tau}$ close to zero, perceiving the signal with little noise. Conversely, an agent skeptical of the Federal Reserve's commitment, perhaps due to concerns about financial stability or labor market conditions, would perceive a much noisier signal, with $\nu^i_{t,\tau}$ deviating significantly from zero. These differences in trust are reflected in the SPF data. Even after the Federal Reserve's long-term target has been shared, disagreement in 5-year PCE forecasts across agents persists, as evidenced by the IQR of forecasts in Figure 2.

The IQR indicates that while disagreement in long-term inflation forecasts gradually decreases following the announcement, it does not completely dissipate. This gradually diminishing (but still existing) disagreement highlights the heterogeneity in agents' reception of publicly accessible signals. Even when exposed to the same information, agents interpret it differently based on their individual trust in the Federal Reserve's credibility, leading to heterogeneous signal reception.

The measurement equation since 2012 can be represented as

$$\begin{pmatrix} S^i_{t,\tau} \\ S^i_{t,\tau\varepsilon} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} \tau_t \\ \varepsilon_t \end{pmatrix} + \begin{pmatrix} \sigma_{\nu,\tau} & 0 \\ 0 & \sigma_{\nu,\tau\varepsilon} \end{pmatrix} \begin{pmatrix} \nu^i_{t,\tau} \\ \nu^i_{t,\tau\varepsilon} \end{pmatrix}. \tag{17}$$

Here, $\sigma_{\nu,\tau\varepsilon}$ and $\sigma_{\nu,\tau}$ represent the magnitudes of the noise terms. The noise terms $\nu^i_{t,\tau\varepsilon}$ and $\nu^i_{t,\tau}$ are both assumed to follow a standard normal distribution, $\mathbb{N}(0,1)$, and $\sigma_{\nu,\tau}$ captures the magnitude of noise in the trend signal. I will refer to the signal $S^i_{t,\tau}$ as the 'trend signal'. Note that the signal structure represents each forecaster's perceived model of π_t, which is not necessarily the same as the true data-generating process.



Note: The red line represents the interquartile range of 5-year PCE forecasts. To capture a wider range of forecasts, the blue dashed line shows the difference between the 90th and 10th percentiles of 5-year PCE forecasts. The gap between forecasts ranked at the 90th and 10th percentiles among survey participants has significantly narrowed since 2012.

Figure 2: Belief Dispersion of 5-Year-Ahead PCE Forecasts

5.3 Smooth Diagnostic Expectations

Given the state-space model, individuals update their information sets and beliefs. The true densities conditional on the current information set and the reference information set obtained in the preceding period are

$$f(\hat{\tau}_{t+h}, \hat{\varepsilon}_{t+h} | \mathscr{I}_t^i) = \mathbb{N}\left(\begin{matrix} \tau^i_{t+h|t} \\ \varepsilon^i_{t+h|t} \end{matrix}, \Sigma_{t+h|t} \right)$$

$$f(\hat{\tau}_{t+h}, \hat{\varepsilon}_{t+h} | \mathscr{I}_t^{i,ref}) = \mathbb{N}\left(\begin{matrix} \tau^i_{t+h|t-1} \\ \varepsilon^i_{t+h|t-1} \end{matrix}, \Sigma_{t+h|t-1} \right)$$

where $\tau^i_{t+h|t} = \mathbb{E}^i_t(\tau_{t+h})$ and $\varepsilon^i_{t+h|t} = \mathbb{E}^i_t(\varepsilon_{t+h})$ represent individual i's Bayesian rational expectations, unaffected by the heuristic. Instead of applying these true densities, forecasters use a distorted density, defined as

$$f^{\theta}(\hat{\tau}_{t+h}, \hat{\varepsilon}_{t+h} | \mathscr{I}_t^i) = f(\hat{\tau}_{t+h}, \hat{\varepsilon}_{t+h} | \mathscr{I}_t^i) \left[\frac{f(\hat{\tau}_{t+h}, \hat{\varepsilon}_{t+h} | \mathscr{I}_t^i)}{f(\hat{\tau}_{t+h}, \hat{\varepsilon}_{t+h} | \mathscr{I}_t^{i,ref})} \right]^{\theta} \frac{1}{Z} \tag{18}$$

where Z is a constant of integration, and θ is assumed to be greater than zero ($\theta > 0$). If $\theta = 0$, this implies that forecasts are formed without distortion, in a fully rational manner. A key concept here is representativeness,

$$rep(\hat{\tau}_{t+h}, \hat{\varepsilon}_{t+h}) \equiv \frac{f(\hat{\tau}_{t+h}, \hat{\varepsilon}_{t+h} | \mathscr{I}_t^i)}{f(\hat{\tau}_{t+h}, \hat{\varepsilon}_{t+h} | \mathscr{I}_t^{i,ref})},$$

which measures the extent to which a forecaster, when faced with current news, subjectively assigns higher or lower likelihoods to future outcomes $(\hat{\tau}_{t+h}, \hat{\varepsilon}_{t+h})$ relative to past reference information. This process triggers selective recall, with possible future outcomes of higher relative frequency being recalled more strongly. When $\theta = 0$, the heuristic does not influence expectations, and forecasts are based purely on the objective conditional probability $f(\hat{\tau}_{t+h}, \hat{\varepsilon}_{t+h} | \mathscr{I}_t^i)$. Notably, $rep(\hat{\tau}_{t+h}, \hat{\varepsilon}_{t+h})$ is affected not only by changes in the conditional mean but also by changes in the conditional variance— by shifts in $\Sigma_{t+h|t}$ and $\Sigma_{t+h|t-1}$, which measure the uncertainty of the current distribution with respect to the reference distribution —when the information set is updated. As beliefs are updated, the ratio of conditional uncertainties, denoted by $R_{t+h|t,t-1}$, plays a crucial role in the smooth diagnostic expectations formation process. To account for this adjustment in conditional uncertainty in relation to the severity of distortion, the effective distortion parameter $\tilde{\theta}_{t,t-1}$ is adopted.

Proposition 3. *Let the reference group of variables, τ_t and ε_t, be given for the period immediately preceding the current one. The ratio of the conditional variance matrices between the current period, t, and the reference period, $t-1$, is defined as a 2-by-2 matrix given by*

$$R_{t+h|t,t-1} \equiv \Sigma_{t+h|t} \Sigma_{t+h|t-1}^{-1}.$$

If $R_{t+h|t,t-1} < \{\frac{1+\theta}{\theta}\} I$, where I is the 2-by-2 identity matrix, the smooth DE density is normally distributed with the conditional mean before 2012 expressed as

$$
\begin{aligned}
\mathbb{E}_t^{i,\theta}(\pi_{t+h}) &= \begin{pmatrix} 1 & \rho_\varepsilon^h \end{pmatrix} \begin{pmatrix} \tau_{t|t}^{i,\theta} \\ \varepsilon_{t|t}^{i,\theta} \end{pmatrix} \\
&= \begin{pmatrix} 1 & \rho_\varepsilon^h \end{pmatrix} \left[\begin{pmatrix} \tau_{t|t-1}^{i} \\ \varepsilon_{t|t-1}^{i} \end{pmatrix} + \left(I + \tilde{\theta}_{t,t-1}\right) K_t \left(S_{t,\tau\varepsilon}^i - \tau_{t|t-1}^i - \varepsilon_{t|t-1}^i \right) \right] \\
&= \begin{pmatrix} 1 & \rho_\varepsilon^h \end{pmatrix} \left[\begin{pmatrix} \tau_{t|t-1}^{i} \\ \varepsilon_{t|t-1}^{i} \end{pmatrix} + \left(I + \tilde{\theta}_{t,t-1}\right) K_t \left(S_{t,\tau\varepsilon}^i - \begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} \tau_{t|t-1}^{i} \\ \varepsilon_{t|t-1}^{i} \end{pmatrix} \right) \right]
\end{aligned}
$$

and the conditional mean after 2012

$$
\begin{aligned}
\mathbb{E}_t^{i,\theta}(\pi_{t+h}) &= \begin{pmatrix} 1 & \rho_\varepsilon^h \end{pmatrix} \begin{pmatrix} \tau_{t|t}^{i,\theta} \\ \varepsilon_{t|t}^{i,\theta} \end{pmatrix} \\
&= \begin{pmatrix} 1 & \rho_\varepsilon^h \end{pmatrix} \left[\begin{pmatrix} \tau_{t|t-1}^{i} \\ \varepsilon_{t|t-1}^{i} \end{pmatrix} + \left(I + \tilde{\theta}_{t,t-1}\right) K_t \left(\begin{pmatrix} S_{t,\tau}^i \\ S_{t,\tau\varepsilon}^i \end{pmatrix} - \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} \tau_{t|t-1}^{i} \\ \varepsilon_{t|t-1}^{i} \end{pmatrix} \right) \right]
\end{aligned}
$$

where effective distortion matrix $\tilde{\theta}_{t,t-1} = \theta R_{t+h|t,t-1}\left(I + \theta(I - R_{t+h|t,t-1})\right)^{-1}$. The variance is formulated as

$$
\begin{aligned}
\mathbb{V}_t^{\theta}(\pi_{t+h}) &= \Sigma_{t+h|t}\left((1+\theta)I - \theta\Sigma_{t+h|t}\Sigma_{t+h|t-1}^{-1}\right)^{-1} \\
&= \Sigma_{t+h|t}\left(I + \theta(I - R_{t+h|t,t-1})\right)^{-1}.
\end{aligned}
$$

Proof. See Appendix G. □

The Kalman gain matrix, K_t, changes over time. Since 2012, with the addition of a new signal, the K_t matrix shifts from a 2-by-1 to a 2-by-2 matrix structure. The reduction in uncertainty, $R_{t+h|t,t-1}$, takes the form of a 2-by-2 matrix throughout all periods[10]. $R_{t+h|t,t-1} < \{\frac{1+\theta}{\theta}\}I$ guarantees the variance of the resulting distorted normal distribution is finite and positive.

[10]In Bianchi, Ilut and Saijo (2024), $R_{t+h|t,t-1}$ is defined as 'the ratio of conditional uncertainty,' which can rise or fall as the information set is updated. In particular, during an uncertainty shock—when updated information becomes more uncertain— $R_{t+h|t,t-1}$may increase. However, in my setting, I assume that as information is updated and the latest news is incorporated, uncertainty in the information set decreases, based on the assumption of a stable economy. For simplicity, I refer to $R_{t+h|t,t-1}$ as the reduction in uncertainty.

Proposition 4. *The effective distortion matrix* $\tilde{\theta}_{t,t-1}$ *decreases in a reduction in uncertainty* $R_{t+h|t,t-1}$.

$$\frac{\partial \tilde{\theta}_{t,t-1}}{\partial R_{t+h|t,t-1}} > 0$$

Proof. See Appendix H. □

In the smooth DE model, $\tilde{\theta}_{t,t-1}$ is positively associated with the ratio of variances $R_{t+h|t,t-1}$. The distortion parameter θ captures the degree to which the diagnostic density inflates the probability of representative states and is constant. However, the *effective* degree of this amplification—represented by $\tilde{\theta}_{t,t-1}$ —varies over time, as it is scaled by $R_{t+h|t,t-1}$. Thus, if $\Sigma_{t+h|t}$ is much smaller than $\Sigma_{t+h|t-1}$, due to highly precise news in the current period, the *effective* magnitude of distortion declines as $R_{t+h|t,t-1}$ decreases. It directly relates to how excessively news influences agent's forecasts. Therefore the first moment of smooth DE density, $\mathbb{E}_t^{i,\theta}(\pi_{t+h})$, is influenced by $R_{t+h|t,t-1}$ adjusting $\tilde{\theta}_{t,t-1}$. In practice, the Federal Reserve's explicit communication about the 2% inflation target in early 2012 significantly contributed to reducing uncertainty surrounding the trend component τ_t which is embedded in $\Sigma_{t+h|t}$. This reduction in uncertainty is particularly sizable following the Federal Reserve's first statement in 2012. This clarity reduces forecasters' reliance on selectively recalled reference information when estimating the trend component.

Moreover, subjective uncertainty, denoted by $\mathbb{V}_t^{\theta}(\pi_{t+h})$, is tied to the ratio of conditional variances, $R_{t+h|t,t-1}$, implying that reduced uncertainty also diminishes subjective uncertainty in forecasts. Forecasters experience a reduction in uncertainty of the current distribution with respect to the reference distribution as incoming news delivers more precise information. Thus, they overstate how precise their updated belief is. This leads to lower uncertainty surrounding their point forecasts, higher confidence in their forecasts. This relationship is evident in the SPF data, where a notable decline in subjective uncertainty is observed following the Federal Reserve's communication in 2012. Consequently, forecasters base their estimates on clearer, current information, reducing reliance on the representativeness heuristic and imperfect memory recall, resulting in smaller distortions in belief updates.

Corollary 1. *As* $\tilde{\theta}_{t,t-1}$ *decreases, heterogeneity across individual forecasts decreases because less weight is given to signals that induce heterogeneity in the information individuals re-*

ceive.

Additionally, a reduction in uncertainty leads to less dispersion across inflation forecasts. The effective distortion parameter, $\tilde{\theta}_{t,t-1}$, serves as an amplifying factor for news. Forecast heterogeneity arises only from the heterogeneous signals that agents receive. When the amplifying factor decreases, each agent places less weight on news in forming their inflation expectations. Consequently, individual forecasts become more aligned with rational expectations as they become less sensitive to heterogeneous signals, thereby reducing disagreement among agents.

6 Estimation

The model is estimated in two stages. In the first stage, I estimate the parameters governing the law of motion in inflation using GDP Price Index data from 1990Q2 to 2021Q4. With these estimates, I proceed to estimate the distortion parameter and the magnitude of signal noises using the simulated method of moments (SMM). This two-step approach addresses the difficulty of estimating all parameters simultaneously through SMM, especially given the presence of latent variables, which complicates the selection of appropriate target moments. To overcome this, I first apply Bayesian estimation to pin down the parameters related to the law of motion, and subsequently use SMM to estimate the distortion parameter and signal noise.

6.1 Bayesian Estimation

Assuming that agents share true values for ρ_ε, γ and σ in the equation (15), I use Bayesian estimation along with a state space model to estimate the parameters ρ_ε, γ, and σ. In the state space model, a transition equation is same as equation (15) and the measurement equation is

$$y_t = \begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} \tau_t \\ \varepsilon_t \end{pmatrix} \tag{19}$$

where y_t represents realized inflation from Philadelphia Fed's Real-Time Dataset for Macroeconomists.

The parameter estimates are reported in Table 2. I report mean posterior estimates, along with the 90% posterior interval. I generate 100,000 draws using the Metropolis–

Parameters	Prior	Posterior Mean	Std. Error	Posterior Distribution (90%)
ρ_ε	$\mathbb{N}$	0.377	0.105	[0.204, 0.551]
γ	$\mathbb{B}$	0.701	0.108	[0.509, 0.867]
σ^2	$\mathbb{IG}$	0.237	0.028	[0.195, 0.286]

Table 2: Estimated Parameters

Hastings algorithm and discard the first 10% as initial burn-in. Further methodological details are presented in the Appendix F.

6.2 Simulated Method of Moments

The advantage of using SMM lies in its flexibility. SMM is highly flexible and can be applied to a wide variety of models, including non-linear and dynamic models where traditional estimation methods (e.g., maximum likelihood) may be difficult or impossible to use. Rather than relying on predefined distributions, SMM leverages simulated data from the model itself, allowing for greater flexibility in application. Furthermore, while the proposed expectations formation model benefits from simplicity and transparency, it is accompanied by the possibility of misspecification. In such cases, moment-based methods like SMM are generally more reliable than other estimation techniques.

Building on this foundation, I apply SMM to estimate key parameters by aligning the variances of forecast errors and forecast revisions—moments that are both observable in the data and tied closely to the parameters being estimated. According to the law of total variance, the variance of forecast errors can be broken down into two components: (i) the average variance of errors across agents and (ii) the variance over time of consensus errors. The former provides information about size of noise in signals ($\sigma_{v,\tau\varepsilon}, \sigma_{v,\tau}$) while the latter captures the overreaction parameter θ. This reasoning similarly applies to the variance of forecast revisions.

The objective is to estimate parameter values that best align with the variances of forecast errors (FE) and forecast revisions (FR), aggregated across time and agents. I propose a range of possible values for $\theta, \sigma_{v,\tau\varepsilon}$ and $\sigma_{v,\tau}$. The target moments are the variances of FE and FR for PGDP forecasts and the variance of FE for 5-Year PCE Inflation Rate (PCE5YR) forecasts. To identify the optimal parameters, I construct a three-dimensional grid, dividing the range of θ into 13 slices, $\sigma_{v,\tau\varepsilon}$ into 9 slices and $\sigma_{v,\tau}$ into 15 slices. Out of resulting 1,755 combinations $(13 \times 9 \times 15)$, I select the one that minimizes the distance between the variances of simulated and observed FE and FR in the survey data.

During the pre-2012 period, from 1990Q2 to 2011Q4, forecasters receive a signal containing a mixture of information about both τ_t and ε_t , which they use to update their forecasts $\tau^{i,\theta}_{t|t}$ and $\varepsilon^{i,\theta}_{t|t}$. For this period, I minimize the sum of two distances: the distance between the model-implied variance of forecast errors and the variance of observed forecast errors from PGDP inflation forecasts, and the distance between the model-implied variance of forecast revisions and the variance of observed forecast revisions. Long-run inflation forecast errors captured by the PCE5YR data from the SPF are unnecessary over this period, as the trend signal $S^i_{t,\tau}$ begins to play a role in the model starting in 2012.

Beginning in 2012, with the introduction of the long-run inflation target, an additional parameter, $\sigma_{\nu,\tau}$, is incorporated into the model. To accommodate this change, I utilize PCE5YR survey data, which provides long-run inflation forecasts. Accordingly, the minimization objective is adjusted to account for the distance between the variance of simulated long-run inflation forecast errors and the variance of observed long-run inflation forecast errors.

To estimate the model parameters, I employ a two-stage SMM approach. In the first stage, I search for parameter values that minimize the distance between simulated and observed moments.

$$\text{Pre-2012:}(\sigma^2_{FE,PGDP} - \hat{\sigma}^2_{FE,PGDP})^2 + (\sigma^2_{FR,PGDP} - \hat{\sigma}^2_{FR,PGDP})^2 \tag{20}$$

$$\text{Post-2012:}(\sigma^2_{FE,PGDP} - \hat{\sigma}^2_{FE,PGDP})^2 + (\sigma^2_{FR,PGDP} - \hat{\sigma}^2_{FR,PGDP})^2 + (\sigma^2_{FE,PCE5YR} - \hat{\sigma}^2_{FE,PCE5YR})^2 \tag{21}$$

Note that the last term $(\sigma^2_{FE,PCE5YR} - \hat{\sigma}^2_{FE,PCE5YR})^2$ in (21) is incorporated only for the period 2012Q1 to 2021Q4. The parameter space for θ is constrained by $\theta \geq 0$. In the second stage, I compute the empirical covariance of the three moments evaluated at the first-stage parameters $(\theta^{FS}, \sigma^{FS}_{\nu,\tau\varepsilon}, \sigma^{FS}_{\nu,\tau})$, invert it to derive the optimal weighting matrix W, and then estimate the second stage parameters $(\theta^*, \sigma^*_{\nu,\tau\varepsilon}, \sigma^*_{\nu,\tau})$ that minimize the following quadratic form

$$\left(\widetilde{\sigma^2_{FE,PGDP}}, \widetilde{\sigma^2_{FR,PGDP}}, \widetilde{\sigma^2_{FE,PCE5YR}}\right)^\top W \left(\widetilde{\sigma^2_{FE,PGDP}}, \widetilde{\sigma^2_{FR,PGDP}}, \widetilde{\sigma^2_{FE,PCE5YR}}\right) \tag{22}$$

	θ	$\frac{\sigma_{\nu,\tau\varepsilon}}{\sqrt{(1-\gamma)}\sigma}$	$\frac{\sigma_{\nu,\tau\varepsilon}}{\sqrt{\gamma}\sigma}$	$\frac{\sigma_{\nu,\tau\varepsilon}}{\sigma}$	$\frac{\sigma_{\nu,\tau}}{\sigma}$
(1990Q2-2011Q4)					
Mixed signal only	0.956	5.961	3.894	3.260	-
	[0.85, 1]	[4.628, 7.105]	[3.023, 4.640]	[2.53, 3.885]	
Mixed signal&target	0.928	5.66	3.696	3.102	3.762
	[0.65, 1]	[3.736, 7.105]	[2.440, 4.640]	[2.01, 3.554]	[0.456, 9.176]
(1990Q2-2021Q4)					
Mixed signal&target	0.736	7.074	4.614	3.866	2.357
	[0.4, 1]	[4.914, 11.530]	[3.246, 5.03]	[3.05, 4.038]	[1.435, 3.443]

Note: The numbers in square brackets indicate a 90% confidence interval. θ is assumed to lie within the interval [0, 1].

Table 3: SMM Estimates of θ, $\sigma_{\nu,\tau\varepsilon}$ and $\sigma_{\nu,\tau}$

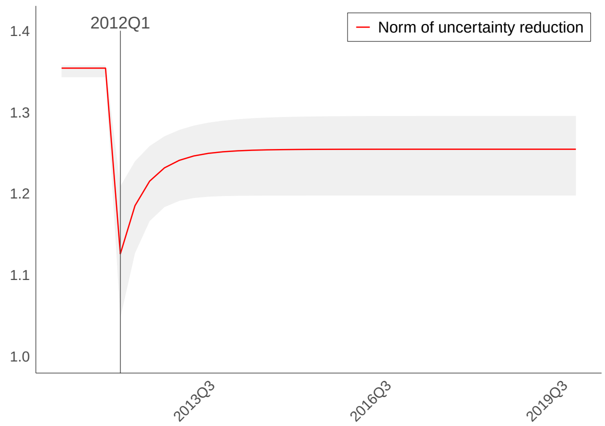
where

$$\widetilde{\sigma^2_{FE,PGDP}} = \sigma^2_{FE,PGDP} - \hat{\sigma}^2_{FE,PGDP}(\theta, \sigma_{\nu,\tau\varepsilon}, \sigma_{\nu,\tau})$$
$$\widetilde{\sigma^2_{FR,PGDP}} = \sigma^2_{FR,PGDP} - \hat{\sigma}^2_{FR,PGDP}(\theta, \sigma_{\nu,\tau\varepsilon}, \sigma_{\nu,\tau})$$
$$\widetilde{\sigma^2_{FE,PCE5YR}} = \sigma^2_{FE,PCE5YR} - \hat{\sigma}^2_{FE,PCE5YR}(\theta, \sigma_{\nu,\tau\varepsilon}\sigma_{\nu,\tau})$$

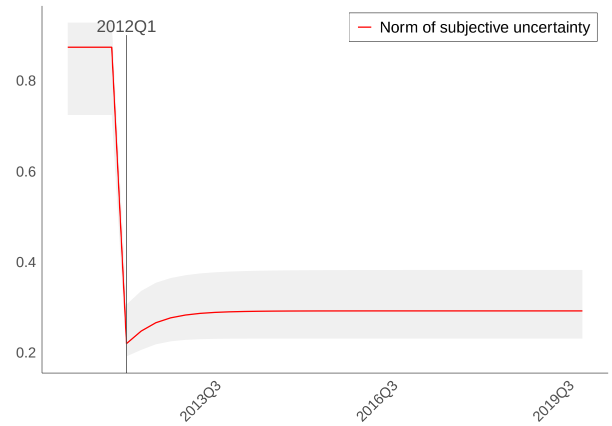
It is important to note that $\widetilde{\sigma^2_{FE,PCE5YR}} = \sigma^2_{FE,PCE5YR} - \hat{\sigma}^2_{FE,PCE5YR}(\theta, \sigma_{\nu,\tau\varepsilon}\sigma_{\nu,\tau})$ is incorporated only for periods since 2012. For the time period between 1990Q2 and 2011Q4, only $\widetilde{\sigma^2_{FE,PGDP}}$ and $\widetilde{\sigma^2_{FR,PGDP}}$ are taken into account. Finally, to construct confidence intervals for the parameter estimates, I perform 200 bootstrap replications.

6.3 Estimation of Parameters

By comparing Smooth DE based solely on the mixed signal with Smooth DE that incorporates both the mixed signal and an additional trend signal – representing the Federal Reserve's statement – it becomes evident that the inclusion of the long-term signal plays a crucial role in reducing the severity of the overreaction in forecasts. Table 3 shows that the value of θ declines from 0.956 during 1990Q2–2011Q4 to 0.736 over 1990Q2–2021Q4, which incorporates both mixed and trend signals. This clearly indicates a weakening in the severity of departure from rational expectations post-2012. To ensure that the decline in θ is not merely a result of introducing an additional parameter for estimation, I



(a) The Frobenius Norm of Uncertainty Reduction



(b) The Frobenius Norm of Subjective Uncertainty

Note: The figure shows the norm of the matrices $R_{t+4|t,t-1}$ (3a) and $\mathbb{V}_t^{\theta}$ (3b), transformed for comparison of their sizes over time. The shaded areas represent the 90% confidence interval. The red line indicates the mean, computed across 200 bootstraps, for each time period.

Figure 3: The Size of Reduction in Uncertainty and Subjective Uncertainty

re-estimate the parameters using data from 1990Q2 to 2011Q4, incorporating not only the variances of FE and FR from four-quarter-ahead inflation forecasts as target moments, but also the variance of FE from long-run forecasts (PCE5YR)[11]. This approach allows me to assess whether adding the variance of long-run forecast errors as a new target moment significantly alters θ and $\sigma_{v,\tau\varepsilon}$ over the period 1990Q2 to 2011Q4. If there is no substantial change compared to estimates that exclude long-run forecast errors, this would suggest that the observed changes in θ and $\sigma_{v,\tau\varepsilon}$ from 1990Q2 to 2021Q4 are primarily driven by the policy change, rather than by the inclusion of the additional target moment. Notably, values for θ and $\sigma_{v,\tau\varepsilon}$ remain largely unchanged, implying that the announcement of the long-term target has a real effect, and that the smaller value of θ is not caused by the inclusion of an additional parameter.

Furthermore, I examine changes in the effective distortion parameter $\tilde{\theta}_{t,t-1}$, the reduction in uncertainty ratio $R_{t+4|t,t-1}$and subjective uncertainty before and after 2012Q1. Turning to the uncertainty ratio, represented as a 2-by-2 matrix, I use the Frobenius norm to compare its magnitude. In the left graph of Figure 3, a decline in the norm of $R_{t+4|t,t-1}$ is observed starting in 2012Q1, suggesting that the long-term inflation goal had an immediate effect in reducing uncertainty. This implies that the posterior variance from in-

[11]The PCE5YR forecast survey only began in 2007. Therefore, I include the total variance of PCE5YR forecast errors over the period 2007Q1–2011Q4.

2011Q4	2012Q1	2012Q2	2012Q3
$\begin{pmatrix} 0.599 & -0.188 \\ -0.000 & 0.712 \end{pmatrix}$	$\begin{pmatrix} 0.513 & -0.145 \\ -0.000 & 0.713 \end{pmatrix}$	$\begin{pmatrix} 0.545 & -0.137 \\ -0.000 & 0.713 \end{pmatrix}$	$\begin{pmatrix} 0.559 & -0.133 \\ -0.000 & 0.712 \end{pmatrix}$

Note: For each of the matrices, the element at [1,1] reflects how much $\tau^{i,\theta}_{t|t}$overreacts (or underreacts) to news about τ_t. Similarly, the element at [1,2] indicates how much $\tau^{i,\theta}_{t|t}$ overreacts (or underreacts) to news about ε_t. The element at [2,1] measures how much $\varepsilon^{i,\theta}_{t|t}$ overreacts (or underreacts) to news about τ_t, while the element at [2,2] captures the extent to which $\varepsilon^{i,\theta}_{t|t}$ overreacts (underreacts) to news about ε_t. A positive value indicates overreaction, while a negative value indicates underreaction. Each element of the matrices is the mean computed across 200 bootstraps.

Table 4: Effective Distortion Matrix $\tilde{\theta}_{t,t-1}$ After 2012

formation updates decreases compared to the prior variance before receiving new information as soon as the announcement is publicized. From 2014Q4 onward, this ratio converges and stabilizes at a lower level than pre-2012 levels. This reduction is primarily driven by a significant decline in uncertainty about the trend component. Across 200 bootstrap samples, the first element of $R_{t+4|t,t-1[1,1]}$, which captures the reduction in conditional posterior uncertainty around trend τ_t relative to conditional prior uncertainty before the information update, shows an average reduction of approximately 7.8% between 2011Q4 and 2012Q1. By contrast, the variance ratio reduction for the cyclical component ε_t, $R_{t+4|t,t-1[2,2]}$, remains consistently around 0.99, indicating that the decline in uncertainty for the cyclical component due to information updates is minimal, regardless of the presence of the additional signal. As shown in the right graph of Figure 3, subjective uncertainty $\mathbb{V}^{\theta}_t$ also declines alongside $R_{t+4|t,t-1}$ from 2012Q1, aligning with empirical evidence from survey data.

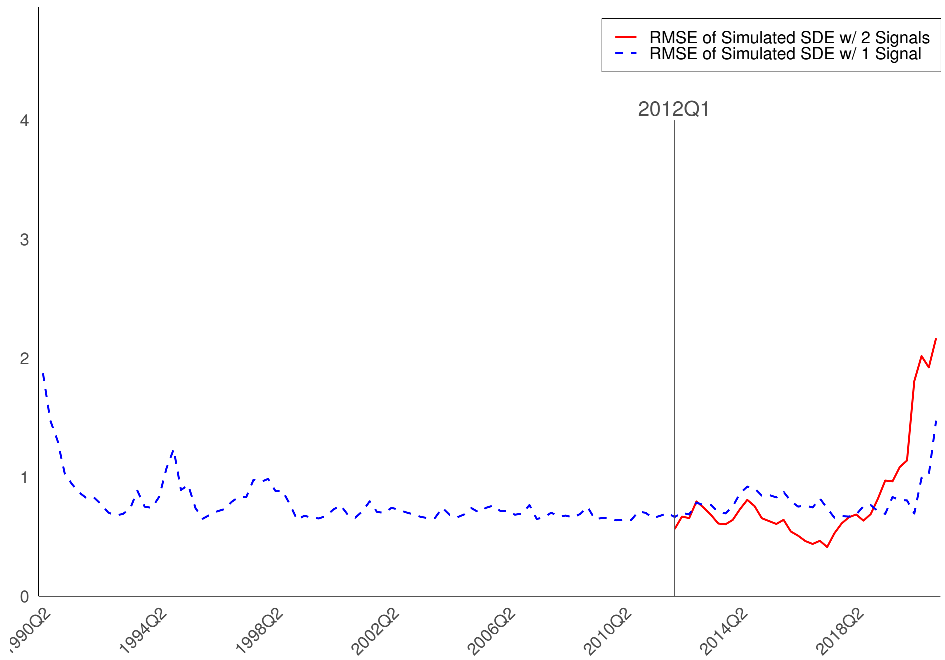
The effective distortion, $\tilde{\theta}_{t,t-1}$, reflects how the reduction in uncertainty affects overreaction to signals. Since $\tilde{\theta}_{t,t-1}$ is a 2-by-2 matrix, an element-wise comparison is required. Table 4 presents the effective distortion matrix around 2012Q1. In forecasting $\tau^{i,\theta}_{t|t}$, the degree of overreaction responding to news decreases since 2012, whereas in forecasting $\varepsilon^{i,\theta}_{t|t}$, there is little change in the degree of overreaction before and after 2012. A closer examination reveals that the overreaction in the belief updating process for $\tau^{i,\theta}_{t|t}$ naturally divides into two parts: 1) reaction to new information about the trend component and 2) reaction to new information about the cyclical component. The overreaction triggered by news regarding τ_t, captured by $\tilde{\theta}_{t,t-1[1,1]}$, clearly diminishes after 2012, suggesting that the announcement plays a role in making trend forecasts more rational. Interestingly, when it comes to news related to the cyclical component, captured by $\tilde{\theta}_{t,t-1[1,2]}$, individuals' forecasts consistently underreact to news both before and after 2012 ($\tilde{\theta}_{t,t-1[1,2]} < 0$). This suggests that, while individuals tend to overreact to news about the trend component when updating their beliefs, $\tau^{i,\theta}_{t|t}$, they counterbalance this by underreacting to news about the cyclical component, thereby helping to stabilize their long-term trend forecasts.

Moreover, there is no substantial difference in the severity of overreaction in terms of expectations regarding the cyclical component ε_t before and after 2012. The overreaction pattern of $\varepsilon^{i,\theta}_{t|t}$ can also be divided into 1) reaction to new information about the trend component, and 2) reaction to new information about the cyclical component. Forecasters clearly overreact to news about the cyclical component ($\tilde{\theta}_{t,t-1[2,2]} > 0$), and the magnitude of this overreaction does not change across the pre- and post-2012 periods. Interestingly, there is neither overreaction nor underreaction of forecasts $\varepsilon^{i,\theta}_{t|t}$ to news about the trend in either period. The element $\tilde{\theta}_{t,t-1[2,1]}$ which measures the extent to which forecasts of the cyclical component ε_t overreact to news about the trend, remains near zero both before and after 2012. This implies that when updating forecasts $\varepsilon^{i,\theta}_{t|t}$, individuals rationally adjust their forecasts even in the context of trend-related information, regardless of their awareness of government policy goals.

In conclusion, the evidence strongly suggests that the public announcement of the long-term inflation target reduces the extent of overreacting expectations related to the trend by lowering conditional variance. This, in turn, leads to greater confidence in forecasts, as reflected by a reduction in subjective uncertainty. However, the overreaction of the cyclical component remains largely unaffected.

6.4 Simulations

Building on the estimation results discussed earlier, I conduct simulations to explore what would have happened if there had been no policy change in 2012, meaning agents would have continued to receive only the mixed signal while the actual data remained unchanged. I assume a panel of 1,000 hypothetical agents predicting four-quarter-ahead inflation under two scenarios. In the first scenario, agents receive both the mixed signal and the trend signal since 2012. Using the parameters $\theta = 0.736$, $\frac{\sigma_{\nu,\pi}}{\sigma} = 2.357$, and $\frac{\sigma_{\nu,\tau\varepsilon}}{\sigma} = 3.866$, the agents form forecasts over the periods from 1990Q2 to 2021Q4. In the second, counterfactual scenario, the agents rely solely on the mixed signal, without receiving the long-term inflation target after 2012. For this scenario, the distortion parameter $\theta = 0.956$, and the mixed signal generated in the first scenario is applied over the period 1990Q2 to 2021Q4. This implies that, since 2012, the difference between the two lines in Figure 4 is driven

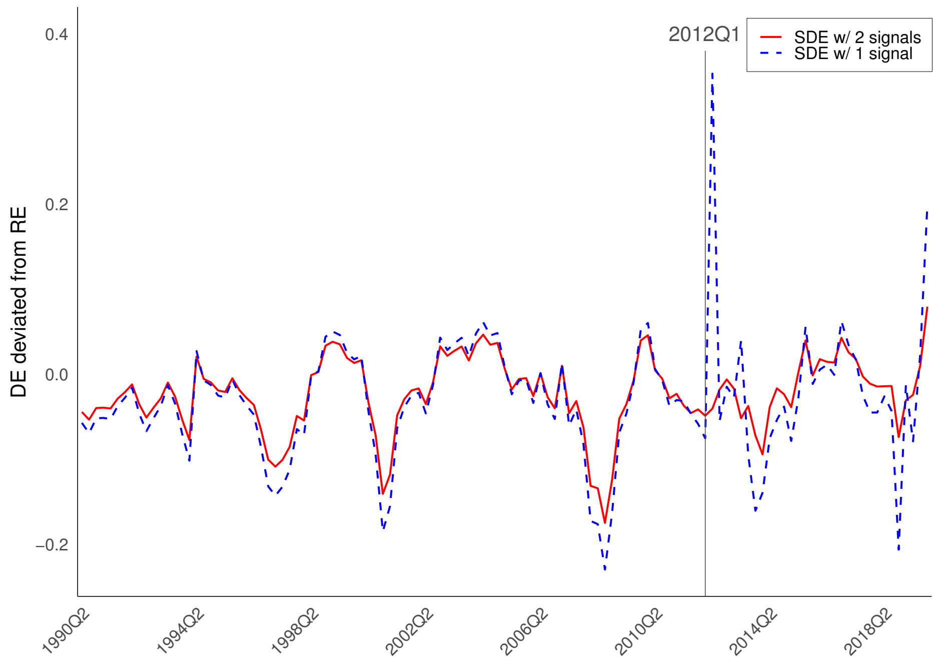


Note: The root mean squared error (RMSE) is calculated across 1,000 samples for each period. For actual data, I use the median forecast from the SPF, so $RMSE_t = \sqrt{\frac{\sum_{i=1}^{1000}(simulated\,SDE_{i,t} - med.SPF_t)^2}{1000}}$. This figure shows the time-varying $RMSE_t$: the red line represents the RMSE of the simulation with two signals, while the blue dashed line indicates the RMSE with only one signal after 2012. A lower RMSE indicates closer alignment with actual forecasts.

Figure 4: Simulated Four-Quarter-Ahead Inflation Forecasts

solely by the trend signal.

The comparison between these two simulation scenarios highlights the impact of receiving an additional signal on inflation forecasts. Figure 4 presents the root mean squared error (RMSE) of simulated Smooth DE under both scenarios, illustrating which simulation aligns more closely with surveyed forecasts. Despite using different θ values in each scenario, prior to 2012, the Smooth DE with one signal and the Smooth DE with two signals display similar explanatory power. After 2012, however, the Smooth DE with two signals more closely fits the median SPF data, suggesting that this estimation better captures the formation of actual expectations. Following the onset of Covid-19, the RMSE under the two-signal case rapidly increases, illustrating a growing divergence between simulated Smooth DE and the actual forecasts reported in the survey. In reality, the fundamental shock to the economy may have diminished trust in the Federal Reserve's messaging, leading forecasters to place less weight on direct information from the Federal Reserve about



Note: The red line represents the magnitude of deviation of the simulated SDE with 2 signals from rational expectations (RE), while the blue dashed line indicates the extent to which the simulated SDE with one mixed signal deviate from RE.

Figure 5: The Extent of DE Deviation from RE

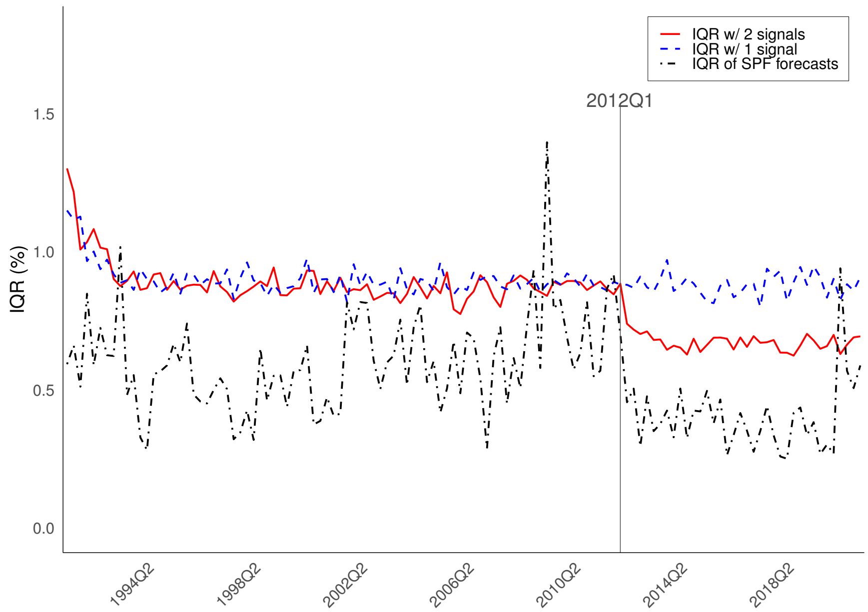
trend inflation. Instead, forecasters may have increasingly relied on the single mixed signal, adjusting their belief-updating behavior as though they were receiving only one signal. Consequently, since 2019, the counterfactual scenario where forecasters receive only the mixed signal might more accurately reflect actual forecasts observed in the SPF. Additionally, in August 2020, the Federal Reserve's adoption of Flexible Average Inflation Targeting, which shifted monetary policy toward a more lenient stance rather than strictly targeting 2% inflation, may have made the Federal Reserve's messages seem somewhat vague or less direct to recipients.

Figure 5 further illustrates that including two signals significantly reduces the deviation from RE. The deviation from RE is calculated using the following formula [12]

$$\text{deviation} = \frac{Smooth\,DE - RE}{RE},$$

and the results are averaged across the 1,000 panelists and presented in Figure 5. The

[12]Rational expectations (RE) are calculated under the assumption that $\theta = 0$.



Note: The red line shows the interquartile range for each period based on forecasts with two signals received since 2012, while the blue dashed line represents the interquartile range under a counterfactual scenario in which agents, after 2012, continue to receive only one mixed signal.

Figure 6: Belief Dispersion of 1-Year Ahead Simulated Inflation Forecasts

graph reveals that deviations are similar before 2012 across both scenarios, but post-2012, the scenario with only one signal becomes increasingly volatile. These findings suggest that sharing a long-term inflation target with the public brings individuals' expectations closer to rational expectations, thereby limiting over-reaction.

In addition, the analysis of forecast dispersion, as shown in Figure 6, demonstrates that sharing a longer run target decreases disagreement among forecasters. The heterogeneity in expectations is primarily driven by information frictions, specifically by the heterogeneous signals that forecasters receive. If the Federal Reserve provides a transparent signal regarding a long-term trend, individuals' information sets will contain less uncertainty as they update them. With this current-period news, individuals recognize that the updated information is more accurate, prompting them to rely less on past memories and more on the true density conveyed by the current news. Consequently, representativeness, measured with respect to reference information, diminishes in its influence on belief updates, reducing the tendency for overreaction to heterogeneous news across agents. As the effect of heterogeneous signals on expectations formation decreases, disagreement among forecasts also declines. This aligns with previous studies showing that well-anchored in-

Parameters	Prior	Posterior Mean	Std. Error	Posterior Distribution (90%)
ρ_ε	$\mathbb{B}$	0.554	0.116	[0.345, 0.726]
γ	$\mathbb{B}$	0.739	0.110	[0.542, 0.904]
σ^2	$\mathbb{IG}$	0.195	0.023	[0.159, 0.235]

Table 5: Estimated Parameters

flation expectations are typically associated with lower dispersion in individual forecasts (Naggert, Rich and Tracy, 2023; Brito, Carriere-Swallow and Gruss, 2018; Ehrmann, 2018; Dovern, Fritsche and Slacalek, 2012).

To measure disagreement, I use the IQR of point forecasts following the methods of Abel et al. (2016), Glas and Hartmann (2016) and Lahiri and Sheng (2010). Figure 6 shows a noticeable decrease in the dispersion of four-quarter-ahead inflation forecasts after 2012, which aligns with the observed SPF data. This reduction in dispersion likely stems from a decrease in disagreement among forecasts about the trend component.

7 Robustness

I explored SMM estimates and analyzed changes and evolving patterns in subjective uncertainty, reduction in uncertainty, and the effective distortion parameter. These analyses build on fundamental parameters driving inflation dynamics, which were estimated through Bayesian estimation. However, the SMM estimates and simulations may be sensitive to the specific parameter values obtained from the Bayesian estimation. To assess robustness, I use alternative parameters derived from different prior distributions. If the new SMM estimates replicate the observed changes and evolving patterns in all three dimensions—subjective uncertainty, reduction in uncertainty, and the effective distortion parameter—it supports the model's validity.

Since the share of the inflation shock attributed to the cyclical component, γ, must lie between 0 and 1, and the volatility of the fundamental shock, σ, must be greater than zero, the priors for these parameters remain unchanged. However, the prior for ρ_ε is adjusted in this exercise by assuming a beta prior distribution. Table 5 indicates that the posterior mean of ρ_ε increases significantly from 0.377 to 0.554, while σ^2 decreases. Based on these results, I now assess whether alternative fundamental parameters affect the outcomes of the SMM estimation.

As shown in Table 6, the value for θ is 0.741, which is not substantially different from the previous value of 0.747. However, the size of the noise noticeably decreases in both mixed and trend signals. In particular, the noise in the trend signal is remarkably small, suggesting that individuals place a high level of trust in the Federal Reserve's announcements regarding long-term inflation targets. This reflects the fact that individuals heavily weigh the Federal Reserve's statements when updating their beliefs in response to new information. As a result, the ratio of posterior variance to prior variance, or uncertainty reduction, falls sharply in the first quarter of 2012 (Figure 7a).

	θ	$\frac{\sigma_{\nu,\tau\varepsilon}}{\sqrt{(1-\gamma)}\sigma}$	$\frac{\sigma_{\nu,\tau\varepsilon}}{\sqrt{\gamma}\sigma}$	$\frac{\sigma_{\nu,\tau\varepsilon}}{\sigma}$	$\frac{\sigma_{\nu,\tau}}{\sigma}$
(1990Q2-2021Q4)					
Mixed signal&target	0.741	1.643	0.977	0.840	0.338
	[0.4, 1]	[1.105, 1.696]	[0.657, 1.009]	[0.565, 0.867]	[0.236, 0.522]

Note: The numbers in square brackets indicate a 90% confidence interval. θ is assumed to lie within the interval $[0,1]$.

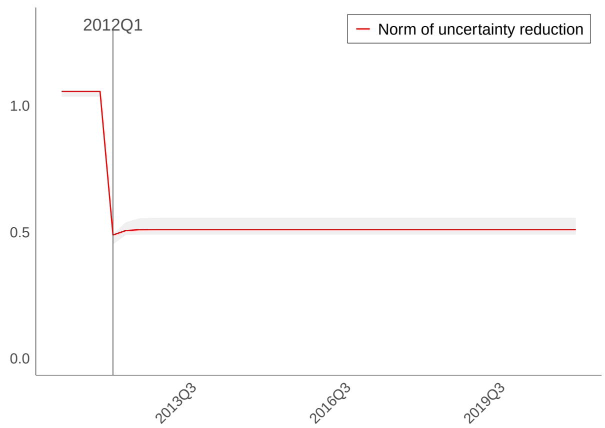
Table 6: SMM Estimates of θ, $\sigma_{\nu,\tau\varepsilon}$ and $\sigma_{\nu,\tau}$

Typically, the largest reduction in uncertainty occurs when the long-run inflation target is initially released, followed by a gradual increase in uncertainty as the effect dissipates over time. However, in this analysis, the high degree of trust in the Federal Reserve's announcements about the trend component lead to a prolonged effect, with uncertainty remaining low. Even after the initial sharp decline, the graph shows only a very slight increase, indicating that the reduction in uncertainty has persisted for an extended period. Consequently, both the subjective uncertainty and the reduction in uncertainty graphs exhibit only minimal increases after 2012Q1, as shown in Figure 7.

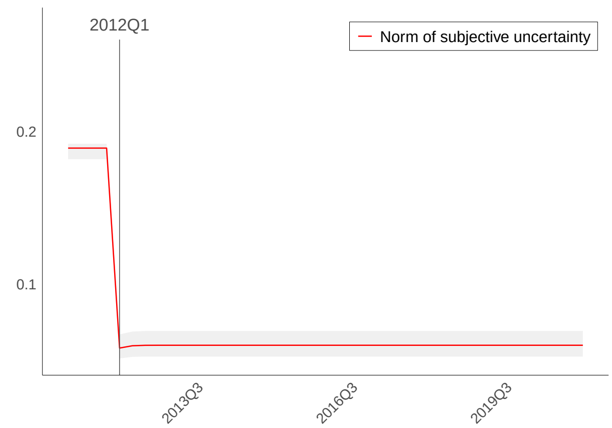
As a result of the Federal Reserve's new policy, individuals rely less on memory and place greater emphasis on current news when forming forecasts, thereby mitigating overreaction.

As shown in Figure 7 the key findings hold consistently, regardless of the parameter values estimated through Bayesian methodology. However, the persistence of the policy's impact depends on the level of trust in the Federal Reserve. The greater the trust, the longer individuals maintain confidence in their beliefs.

In addition, Table 7 shows a significant decline in the element $\tilde{\theta}_{t,t-1[1,1]}$, dropping from 0.436 to 0.022 in 2012Q1. This drop aligns with the pattern observed in previous analysis, reinforcing the idea that the announcement helped bring trend forecasts closer to rational expectations. Similarly, $\tilde{\theta}_{t,t-1[1,2]}$ and $\tilde{\theta}_{t,t-1[2,1]}$ retain their negative signs, in line with the



(a) The Frobenius Norm of Uncertainty Reduction



(b) The Frobenius Norm of Subjective Uncertainty

Note: The figure shows the norm of the matrices $R_{t+4|t,t-1}$(7a) and $\mathbb{V}_t^{\theta}$ (7b), transformed for comparison of their sizes over time. The shaded areas represent the 90% confidence interval. The red line represents the mean, computed across 200 bootstraps, for each time period.

Figure 7: The Size of Reduction in Uncertainty and Subjective Uncertainty

2011Q4	2012Q1	2012Q2	2012Q3
$\begin{pmatrix} 0.436 & -0.302 \\ -0.273 & 0.465 \end{pmatrix}$	$\begin{pmatrix} 0.022 & -0.016 \\ 0.046 & 0.244 \end{pmatrix}$	$\begin{pmatrix} 0.061 & -0.019 \\ -0.032 & 0.249 \end{pmatrix}$	$\begin{pmatrix} 0.063 & -0.019 \\ -0.035 & 0.249 \end{pmatrix}$

Note: For each of the matrices, the element at [1,1] reflects how much $\tau_{t|t}^{i,\theta}$overreacts (or underreacts) to news about τ_t. Similarly, the element at [1,2] indicates how much $\tau_{t|t}^{i,\theta}$ overreacts (or underreacts) to news about ε_t. The element at [2,1] measures how much $\varepsilon_{t|t}^{i,\theta}$ overreacts (or underreacts) to news about τ_t, while the element at [2,2] captures the extent to which $\varepsilon_{t|t}^{i,\theta}$ overreacts (underreacts) to news about ε_t. A positive value indicates overreaction, while a negative value indicates underreaction. Each element of the matrices is the mean computed across 200 bootstraps.

Table 7: Effective Distortion Matrix $\tilde{\theta}_{t,t-1}$ Post-2012

findings reported in Table 4. In contrast to the earlier analysis, $\tilde{\theta}_{t,t-1[2,2]}$ exhibits a noticeable decrease after 2012.

8 Analysis Using a New Keynesian Model

Extending the partial equilibrium setup, I incorporate the smooth diagnostic expectations (DE) framework into a New Keynesian (NK) model to examine whether the Federal Reserve's inflation target announcement contributes to stabilizing realized inflation. The three-equation NK model augmented with diagnostic expectations follows L'Huillier et al. (2023). My model differs from the original in two key respects: (1) agents form smooth diagnostic expectations rather than canonical diagnostic expectations, and (2) agents receive noisy signals about inflation. As described in Section 5, it is assumed that agents infer τ_t and ε_t separately from signals.

$$\bar{y}_t = \mathbb{E}_t[\bar{y}_{t+4}] - (\bar{i}_t - (\mathbb{E}_t^{\theta}[\bar{\pi}_{t+4}] + \theta(\bar{\pi}_t - \mathbb{E}_{t-1}[\bar{\pi}_{t+4}]))) \tag{23}$$

$$\bar{\pi}_t = \beta\mathbb{E}_t^{\theta}[\bar{\pi}_{t+4}] + \kappa(\bar{y}_t - \bar{a}_t) \tag{24}$$

$$\bar{i}_t = \phi_\pi \bar{\pi}_t + \phi_x(\bar{y}_t - \bar{a}_t) \tag{25}$$

where $\kappa \equiv \frac{\epsilon_p - 1}{\psi_p}(1+\nu)$[13], and the aggregate TFP shock processes are given by

$$\bar{a}_t = \rho_a \bar{a}_{t-1} + \varepsilon_{a,t} \tag{26}$$

where $\varepsilon_{a,t} \sim iid\mathbb{N}(0,1)$.

Note that variables with a bar denote log deviations from steady state. Under the closed-economy assumption, $\bar{y}_t = \bar{c}_t$. The expectation operator with superscript θ, $\mathbb{E}^{\theta}$, smooth diagnostic expectations, while the expectation operator without superscript, $\mathbb{E}$, denotes rational expectations. Inflation expectations $(\mathbb{E}_t^{\theta}[\bar{\pi}_{t+4}], \mathbb{E}_{t-1}[\bar{\pi}_{t+4}])$ are formed described in Proposition 3 of Subsection 5.3, and all expectations are subject to noisy information. Since only inflation expectations have been modeled as shaped by smooth

[13] ν is inverse Frisch elasticity, ψ_p is Rotemberg pricing parameter, $\epsilon_p > 1$ is the elasticity of substitution in intermediate good's demand. Each parameter is not separately identified in estimation. More details are found in L'Huillier et al. (2023).

diagnostic expectations so far, output expectations are assumed to follow rational expectations.

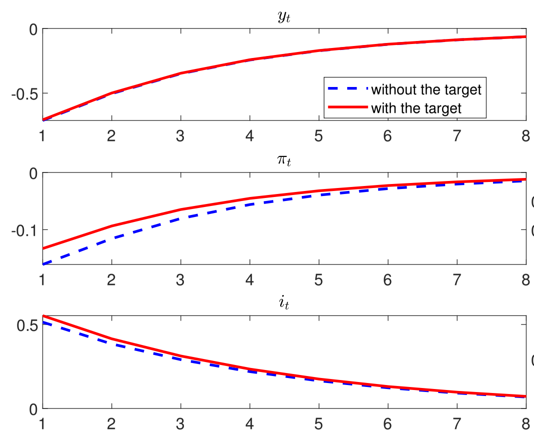
The analysis uses annualized quarterly data for inflation, output growth, and the interest rate, the expected variables ($\mathbb{E}_t[\bar{y}_{t+4}], \mathbb{E}_t^{\theta}[\bar{\pi}_{t+4}], \mathbb{E}_{t-1}[\bar{\pi}_{t+4}]$) are replaced with four-quarters-ahead expectations to align with the Survey of Professional Forecasters (SPF) four-quarters-ahead median forecasts for real GDP (RGDP) and the GDP deflator (PGDP). This specification is motivated by two considerations. First, one-quarter-ahead SPF forecasts are relatively noisy and less informative about agents' perceived policy stance. Second, since this exercise aims to evaluate how transparent communication of the Federal Reserve's long-run inflation target mitigates individuals' overreactive short-term belief updates, matching the model's expectation horizon to the SPF's one-year-ahead forecasts provides a more relevant empirical counterpart. The realized data are obtained from the FRED database[14]. Using the RISE toolbox, I estimate the uncertainty ratio matrix R under the assumption of a regime shift in 2012 and generate impulse response functions for the two regimes. Except for the ratio R, all other parameters are assumed to be non-switching. The estimated variables are reported in Appendix I.

Based on the estimation results, I generate impulse responses of output, inflation, and the interest rate to a cost-push shock, a monetary policy shock, and a demand shock with horizon = 20 quarters (Figure 8). Starting with the monetary policy shock, the log deviation of inflation, $\bar{\pi}_t$,exhibits lower volatility under the regime with the Fed's inflation target, while output barely responds. The reduced volatility of $\bar{\pi}_t$ reflects less overreactive smooth DE, $\mathbb{E}_t^{\theta}[\bar{\pi}_{t+1}]$, as agents anticipate that inflation will revert toward its normal level. Because expectations react less strongly to news and trend inflation expectations is more firmly anchored, realized inflation declines only modestly in response to the shock. Consequently, the nominal interest rate displays a slightly larger deviation, consistent with the Taylor rule's response to a relatively stable inflation path.

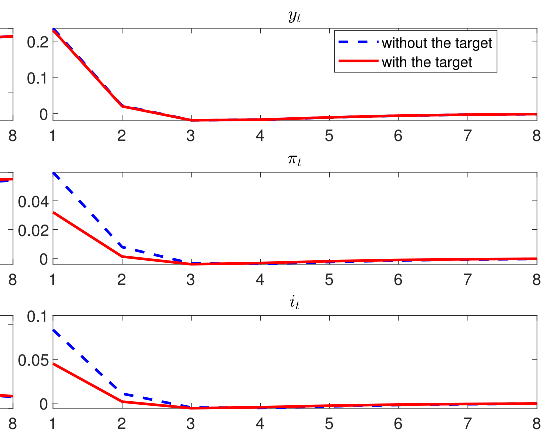
In response to a demand shock, the difference in the log deviation of y_t between the two regimes is negligible because y_t follows rational expectations in this calibration, and the ex-ante real interest rate term remains similar across the two regimes. The gap in the real rate between the regime with the Fed's target and that without the target is less than 0.005, resulting in seemingly identical output responses. In both regimes, however, π_t and i_t show lower volatility under the Fed's inflation target.

In the case of a cost-push shock, smooth diagnostic expectations react less to the dis-

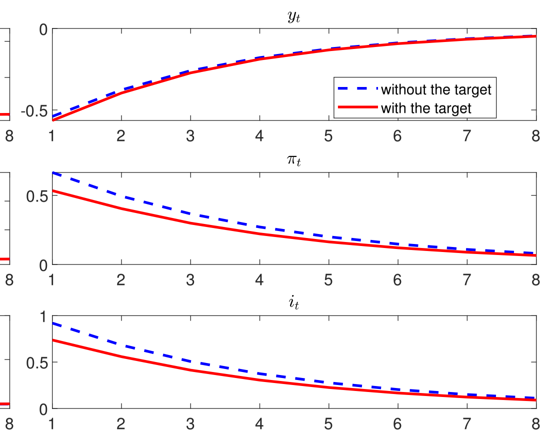
[14]The series names are GDPC1, FEDFUNDS, and GDPDEF, respectively.



(a) monetary policy shock



(b) demand shock



(c) cost-push shock

Note: The red line represents the IRFs under the Fed's target, while the blue dashed line represents the IRFs without the Fed's target, in response to a positive monetary policy shock, demand shock, and cost-push shock. For clarity, the figures display responses over the first eight quarters.

Figure 8: Impulse Responses of output, inflation and interest rate

turbance, thereby reducing the forward-looking component of the Phillips curve. The weaker Phillips amplification of the cost shock yields a smaller and less persistent response of inflation, and consequently, a more muted adjustment in the nominal interest rate through the Taylor rule. Regardless of the shock type, inflation responds with lower volatility under the Fed's inflation target, reflecting the stabilizing role of the Fed's information sharing in shaping inflation expectations.

9 Conclusions

The success of monetary policy hinges on clear and accurate communication of its plans and goals. Given that short-term inflation expectations can influence everyday decisions, such as consumer spending, it is essential to examine whether monetary policy affects short-term inflation forecasts. The key takeaway of this paper is that sharing precise numerical targets with the public not only anchors long-term inflation forecasts but also shapes short-term forecasts in a more rational and less distorted manner. When estimating future states, individuals rely on the representativeness heuristic, assigning greater weight to salient memories rather than objectively assessing probabilities. However, when provided with accurate information, individuals reduce their reliance on subjective recall and form expectations based on more objective likelihood of future outcome delivered, thereby mitigating over-reaction to news. This paper specifically focuses on the 2012 Statement on Longer-Run Goals and Monetary Policy Strategy, which provided concrete information on trend inflation, significantly reducing inflation forecast uncertainty and enhancing individuals' confidence in their forecasts.

Adding such an additional, reliable signal—compared to relying solely on one source—facilitates more rational belief updating and, consequently, reduces disagreement among individuals. While the decrease in long-term inflation forecast dispersion stems from the anchoring effect, the narrowing of short-term inflation forecast dispersion appears to result from lessened over-reaction to incoming information. This shift leads to expectations that align more closely with rational expectations, thereby reducing disagreement.

Moreover, I assume a stable economic environment, contributing to the broader understanding of how policy communication affects expectations in relatively calm periods. However, in times of severe disruptions—such as the Covid-19 pandemic or the war between Russia and Ukraine—subjective uncertainty and effective distortion may rise, particularly if agents doubt the sufficiency of transparent communication during such shocks. Future research could examine the role of fundamental shocks in shaping inflation expectations, specifically assessing how these shocks interact with policy communication strategies and whether these strategies can mitigate heightened subjective uncertainty in turbulent times.

Integrating this expectations-formation framework into a standard three-equation New Keynesian model further shows that announcing the inflation target helps stabilize the responses of realized inflation to various structural shocks. Within this limited framework where, for simplicity, only output expectations are assumed to follow rational expectations, the model demonstrates that the Fed's target announcement effectively reduces agents' overreaction to current news under smooth diagnostic expectations, thereby contributing to the stabilization of realized inflation.

Although this paper includes the Covid-19 period, it treats shocks from these disruptions as drawn from the same distribution as those in normal times. Extending this work could involve exploring policy guidance's role during extreme events modeled with a state-dependent approach, where shocks might come from a different normal distribution with a higher mean and variance. Such a model would capture how extreme shocks influence the degree of over-reaction and the shift in conditional uncertainty. This approach could also shed light on whether the interaction between uncertainty in news and fundamental shocks results in amplification or dampening effects. Understanding whether transparent communication by the Federal Reserve can reduce distortion and curb over-reactive belief adjustments under these conditions would provide valuable insights for policy design in periods of heightened uncertainty.

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Appendices

A Survey of Professional Forecasters

Please fill in forecast of the following U.S. business indicators.

		Quarterly Data						Annual Data [a]				
	L / G	2024:Q1	2024:Q2	2024:Q3	2024:Q4	2025:Q1	2025:Q2	2023	2024	2025	2026	2027
1. Nominal GDP		28284.5						27360.9				
2. GDP Price Index (Chain)		124.24						122.28				
3. Corporate Prof After Tax		.						2672.9				
4. Civilian Unemp Rate	L	3.8						3.6				
5. Nonfarm Payroll Employment [b]		157841						156066				
6. Industrial Prod Index		102.3						102.8				
7. Housing Starts		1.415						1.423				
8. T-Bill Rate, 3-month	L	5.23						5.07				
9. Moody's AAA Corp Bond Yield [c]	L	.						.				
10. Moody's BAA Corp Bond Yield [c]	L	.						.				
11. Treasury Bond Rate, 10-year	L	4.16						3.96				

Note: This question is included in the survey distributed in the second quarter of 2024.

Figure 9: U.S. Business Indicators

B Survey of Professional Forecasters

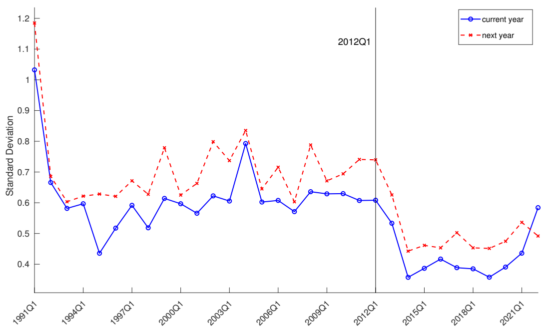
Please indicate what probabilities you would attach to the various possible percentage change (annual-average over annual-average) in the chain-weighted GDP price index. The probabilities of these alternative forecasts should add up to 100.

	Probability of indicated percent change in chain-weighted GDP price index	
	2023-2024	2024-2025
4 percent or more		
3.5 to 3.9 percent		
3.0 to 3.4 percent		
2.5 to 2.9 percent		
2.0 to 2.4 percent		
1.5 to 1.9 percent		
1.0 to 1.4 percent		
0.5 to 0.9 percent		
0.0 to 0.4 percent		
Will decline		
TOTAL	0	0

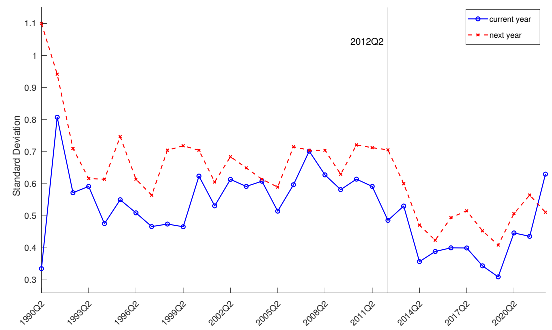
Note: This question is included in the survey distributed in the second quarter of 2024.

Figure 10: Probabilities of Year-Over-Year Changes in the GDP Price Index

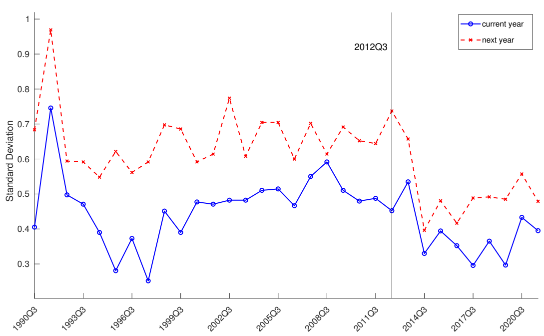
C Subjective Uncertainty in Fixed-Event Inflation Forecasts



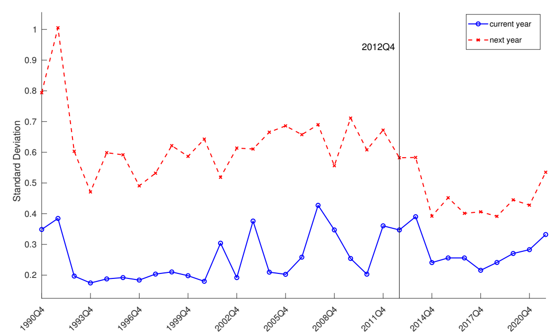
(a) Q1 Vintage



(b) Q2 Vintage



(c) Q3 Vintage

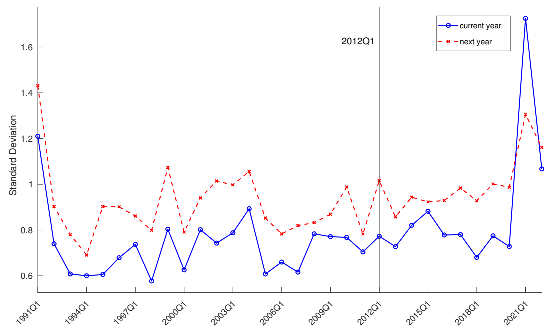


(d) Q4 Vintage

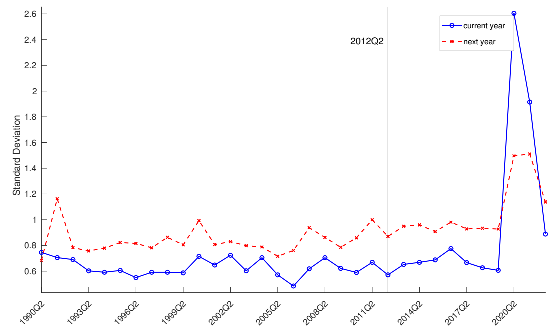
Note: The figure shows subjective uncertainty measured in fixed-event forecasts from the SPF. The blue line with circles depicts the median subjective uncertainty, expressed in standard deviations, for current-year inflation. The red dashed line illustrates the median subjective uncertainty, also expressed in standard deviations, for next-year inflation. A normal distribution is fitted to individual-level survey data, from which the standard deviations are derived.

Figure 11: Inflation Rate

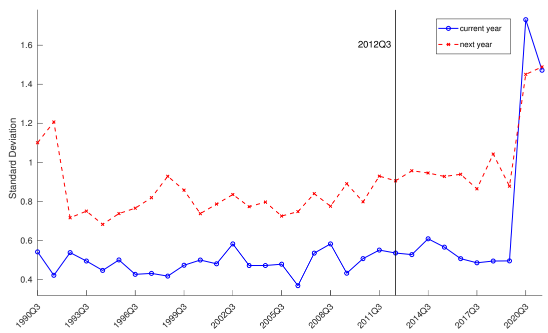
D Subjective Uncertainty in Fixed-Event Non-Inflation Forecasts



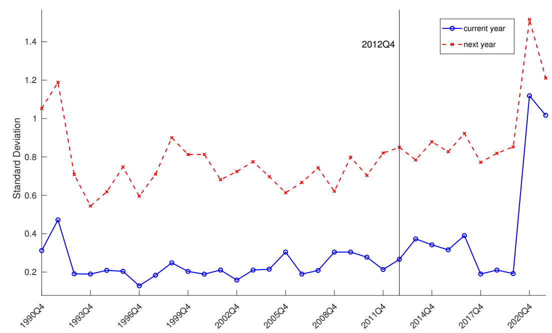
(a) Q1 Vintage



(b) Q2 Vintage



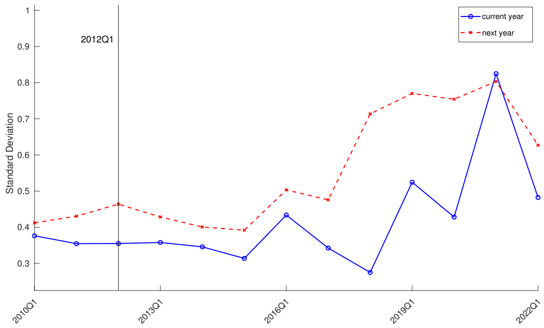
(c) Q3 Vintage



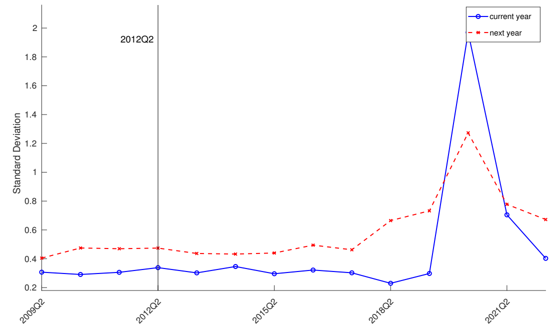
(d) Q4 Vintage

Note: The figure shows subjective uncertainty measured in fixed-event forecasts from the SPF. The blue line with circles depicts the median subjective uncertainty, expressed in standard deviations, for current-year percentage change in real GDP. The red dashed line illustrates the median subjective uncertainty, also expressed in standard deviations, for next-year percentage change in real GDP. A normal distribution is fitted to individual-level survey data, from which the standard deviations are derived.

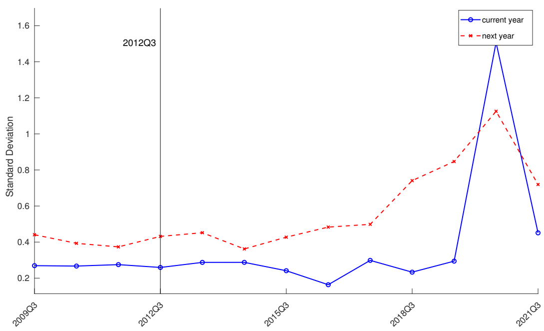
Figure 12: Percentage Change in Real GDP



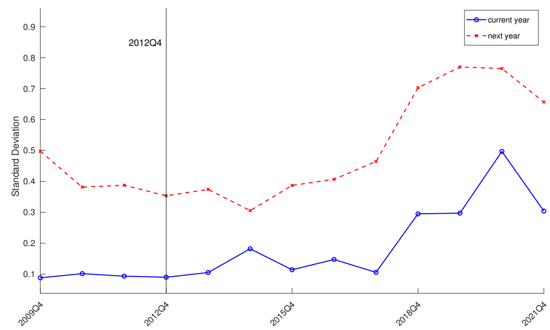
(a) Q1 Vintage



(b) Q2 Vintage



(c) Q3 Vintage



(d) Q4 Vintage

Note: The figure shows subjective uncertainty measured in fixed-event forecasts from the SPF. The blue line with circles depicts the median subjective uncertainty, expressed in standard deviations, for current-year civilian unemployment rates. The red dashed line illustrates the median subjective uncertainty, also expressed in standard deviations, for next-year civilian unemployment rates. A normal distribution is fitted to individual-level survey data, from which the standard deviations are derived.

Figure 13: Unemployment Rate

E CG Tests of Other Macroeconomic Variables

	1990Q2-2011Q4	2012Q1-2022Q1	1990Q2-2022Q1	1990Q2-2011Q4	2012Q1-2022Q1	1990Q2-2022Q1
β_0	-0.022	-0.799	-0.351^{**}	-	-	-
	(0.151)	(0.375)	(0.172)			
β_1	0.225	-0.543^{*}	-0.299	0.094	-0.557^{*}	-0.345
	(0.220)	(0.295)	(0.266)	(0.194)	(0.310)	(0.279)
Obs.	2320	1182	3554	2312	1177	3543
FE	No	No	No	Yes	Yes	Yes

Note: CG test results using IV regression. Obs. indicates the sample size. Robust standard errors are in parentheses;***indicates significance at the 1% level. **indicates significance at the 5% level, and *indicates significance at the 10% level.

(a) Percentage Change in Real GDP

	1990Q2-2011Q4	2012Q1-2022Q1	1990Q2-2022Q1	1990Q2-2011Q4	2012Q1-2022Q1	1990Q2-2022Q1
β_0	0.041	-0.146	0.0231	-	-	-
	(0.094)	(0.297)	(0.127)			
β_1	0.670^{***}	-0.472^{**}	-0.279	0.530^{***}	-0.492^{***}	-0.307
	(0.230)	(0.188)	(0.293)	(0.200)	(0.190)	(0.276)
Obs.	2413	1274	3741	2407	1270	3733
FE	No	No	No	Yes	Yes	Yes

Note: CG test results using IV regression. Obs. indicates the sample size. Robust standard errors are in parentheses;***indicates significance at the 1% level. **indicates significance at the 5% level, and *indicates significance at the 10% level.

(b) Unemployment Rate

Table 8: CG Test Results at Individual Level

F Bayesian Estimation

I assume prior distributions as

$$\rho_\varepsilon \sim \mathbb{N}(\mu_\rho, \sigma_\rho^2)$$
$$\gamma \sim \mathbb{B}(\alpha_\gamma, \beta_\gamma)$$
$$\sigma^2 \sim \mathbb{IG}(\alpha_{\sigma^2}, \beta_{\sigma^2}),$$

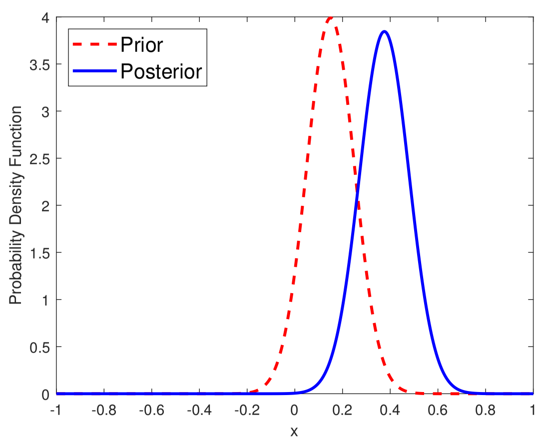
and set hyper-parameters as follows.

hyper-parameters	Value
μ_ρ	0.15
σ_ρ^2	0.01
α_γ	18
β_γ	3
α_{σ^2}	15
β_{σ^2}	11

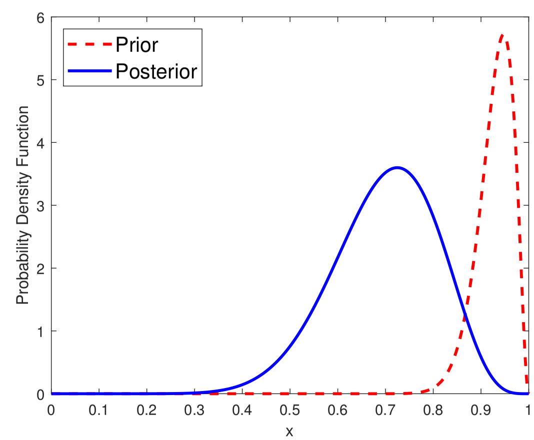
For initial values $x^{(0)} = (\rho_\varepsilon^{(0)}, \gamma^{(0)}, \sigma^{(0)})$, I guess unconditional mean of prior distributions.

A normal prior distribution is selected for ρ_ε, anticipating that isolating the cyclical component after removing the trend in inflation would result in lower persistence of shocks. While the trend component captures long-term patterns, the cyclical component focuses on short-term economic fluctuations. This may cause the autocorrelation coefficient in an AR(1) model to approach zero or even become negative. To account for this potential variability, a normal prior is considered appropriate for ρ_ε. In contrast, γ, representing a share ratio constrained to the interval [0, 1], is modeled using a beta distribution, which is optimal for such bounded parameters. Lastly, given that σ^2 is strictly positive, an inverse-gamma distribution is chosen for its prior. A burn-in period of 10,000 iterations out of 100,000 draws is employed, discarding the initial samples to stabilize the parameters and enhance the reliability of the posterior distribution.

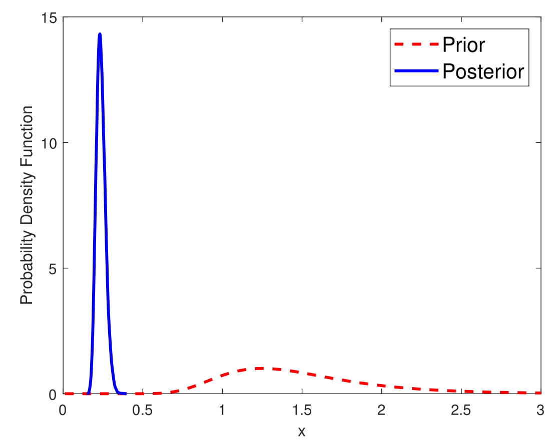
The following figure plots prior and posterior distributions.



(a) Distribution for ρ_ε



(b) Distribution for γ



(c) Distribution for σ^2

G Proof of Proposition 3

We start by rewriting equation (18)

$$f^{\theta}(x_{t+h}) \propto \left[\begin{array}{l} \exp\left(-\frac{1}{2}(x_{t+h}-x^i_{t+h|t})^\top \Sigma^{-1}_{t+h|t}(x_{t+h}-x^i_{t+h|t})\right) \\ \left\{ \frac{\exp\left(-\frac{1}{2}(x_{t+h}-x^i_{t+h|t})^\top \Sigma^{-1}_{t+h|t}(x_{t+h}-x^i_{t+h|t})\right)}{\exp\left(-\frac{1}{2}(x_{t+h}-x^i_{t+h|t-1})^\top \Sigma^{-1}_{t+h|t-1}(x_{t+h}-x^i_{t+h|t-1})\right)} \right\}^{\theta} \quad \frac{1}{Z} \end{array} \right]$$

where $x_{t+h} = \begin{pmatrix} \tau_{t+h} & \varepsilon_{t+h} \end{pmatrix}^\top$ represents the actual realized inflation components, and $x^i_{t+h|t} = \begin{pmatrix} \tau^i_{t+h|t} & \varepsilon^i_{t+h|t} \end{pmatrix}^\top$ denotes individual i's h-ahead inflation forecast for the trend and cyclical components.

Since $\left\{\frac{exp(a)}{exp(b)}\right\}^{\theta} = exp(\theta(a-b))$,

$$f^{\theta}(x_{t+h}) \propto \left[\begin{array}{l} \exp\left(-\frac{1}{2}(x_{t+h}-x^i_{t+h|t})^\top \Sigma^{-1}_{t+h|t}(x_{t+h}-x^i_{t+h|t})\right) \\ \left\{\exp\left(\theta\left\{\left(-\frac{1}{2}(x_{t+h}-x^i_{t+h|t})^\top \Sigma^{-1}_{t+h|t}(x_{t+h}-x^i_{t+h|t})\right)\right.\right.\right. \\ \left.\left.\left. -\left(-\frac{1}{2}(x_{t+h}-x^i_{t+h|t-1})^\top \Sigma^{-1}_{t+h|t-1}(x_{t+h}-x^i_{t+h|t-1})\right)\right\}\right)\right\} \frac{1}{Z} \end{array} \right]$$

$$f^{\theta}(x_{t+h}) \propto \left[\begin{array}{l} \exp\left(-(1+\theta)\frac{1}{2}(x_{t+h}-x^i_{t+h|t})^\top \Sigma^{-1}_{t+h|t}(x_{t+h}-x^i_{t+h|t})\right. \\ \left. +\frac{1}{2}\theta(x_{t+h}-x^i_{t+h|t-1})^\top \Sigma^{-1}_{t+h|t-1}(x_{t+h}-x^i_{t+h|t-1})\right) \frac{1}{Z} \end{array} \right]$$

$$f^{\theta}(x_{t+h}) \propto \left[\begin{array}{l} \exp\left(-\frac{1}{2}\Sigma^{-1}_{t+h|t}\left\{(1+\theta)(x_{t+h}-x^i_{t+h|t})^\top(x_{t+h}-x^i_{t+h|t})\right.\right. \\ \left.\left. -\theta(x_{t+h}-x^i_{t+h|t-1})^\top \Sigma_{t+h|t}\Sigma^{-1}_{t+h|t-1}(x_{t+h}-x^i_{t+h|t-1})\right\}\right) \frac{1}{Z} \end{array} \right]$$

By developing the squared terms and focusing on the terms involving x_{t+h}, we arrive at

$$f^{\theta}(x_{t+h}) \propto \left[\begin{array}{l} \exp\left(-\frac{1}{2}\Sigma^{-1}_{t+h|t}\left\{(1+\theta)I-\theta\Sigma_{t+h|t}\Sigma^{-1}_{t+h|t-1}\right\}\left(x^{\top}_{t+h}x_{t+h}\right.\right. \\ -2(1+\theta)x_{t+h}\left((1+\theta)I-\theta\Sigma_{t+h|t}\Sigma_{t+h|t-1}\right)^{-1}x^{i}_{t+h|t} \\ \left.\left.+2\theta x^{\top}_{t+h}\left((1+\theta)I-\theta\Sigma_{t+h|t}\Sigma_{t+h|t-1}\right)^{-1}\Sigma_{t+h|t}\Sigma_{t+h|t-1}x^{i}_{t+h|t-1}\right)\right) \end{array}\right]$$

This equation represents the kernel of a normal density with the following mean

$$\begin{aligned}
\mathbb{E}^{i,\theta}_{t}(x_{t+h}) &= \left((1+\theta)I-\theta\Sigma_{t+h|t}\Sigma^{-1}_{t+h|t-1}\right)^{-1}\left((1+\theta)x^{i}_{t+h|t}-\theta\Sigma_{t+h|t}\Sigma^{-1}_{t+h|t-1}x^{i}_{t+h|t-1}\right) \\
&= \left((1+\theta)I-\theta R_{t+h|t,t-1}\right)^{-1}\left((1+\theta)x^{i}_{t+h|t}-\theta R_{t+h|t,t-1}x^{i}_{t+h|t-1}\right) \\
&= \left((1+\theta)I-\theta R_{t+h|t,t-1}\right)^{-1}x^{i}_{t+h|t}+\theta\left((1+\theta)I-\theta R_{t+h|t,t-1}\right)^{-1}R_{t+h|t,t-1}(R^{-1}_{t+h|t,t-1}x^{i}_{t+h|t}-x^{i}_{t+h|t-1}) \\
&= \left(I+\theta(I-R_{t+h|t,t-1})\right)^{-1}x^{i}_{t+h|t}+\theta\left(I+\theta(I-R_{t+h|t,t-1})\right)^{-1}R_{t+h|t,t-1}(R^{-1}_{t+h|t,t-1}x^{i}_{t+h|t}-x^{i}_{t+h|t-1}) \\
&= \left(I+\theta(I-R_{t+h|t,t-1})\right)^{-1}x^{i}_{t+h|t}+\theta\left(I+\theta(I-R_{t+h|t,t-1})\right)^{-1}R_{t+h|t,t-1}R^{-1}_{t+h|t,t-1}x^{i}_{t+h|t} \\
&\quad -\theta R_{t+h|t,t-1}\left(I+\theta(I-R_{t+h|t,t-1})\right)^{-1}x^{i}_{t+h|t-1} \\
&= (I+\theta I)\left(I+\theta(I-R_{t+h|t,t-1})\right)^{-1}x^{i}_{t+h|t}-\theta R_{t+h|t,t-1}\left(I+\theta(I-R_{t+h|t,t-1})\right)^{-1}x^{i}_{t+h|t-1}
\end{aligned}$$

where $R_{t+h|t,t-1}=\Sigma_{t+h|t}\Sigma^{-1}_{t+h|t-1}$. Since $I+\theta I = I+\theta(I-R_{t+h|t,t-1})+\theta R_{t+h|t,t-1}$, it follows that

$$\begin{aligned}
\mathbb{E}^{i,\theta}_{t}(x_{t+h}) &= \left(I+\theta(I-R_{t+h|t,t-1})+\theta R_{t+h|t,t-1}\right)\left(I+\theta(I-R_{t+h|t,t-1})\right)^{-1}x^{i}_{t+h|t} \\
&\quad -\theta R_{t+h|t,t-1}\left(I+\theta(I-R_{t+h|t,t-1})\right)^{-1}x^{i}_{t+h|t-1} \\
&= x^{i}_{t+h|t}+\theta R_{t+h|t,t-1}\left(I+\theta(I-R_{t+h|t,t-1})\right)^{-1}x^{i}_{t+h|t}-\theta R_{t+h|t,t-1}\left(I+\theta(I-R_{t+h|t,t-1})\right)^{-1}x^{i}_{t+h|t-1} \\
&= x^{i}_{t+h|t}+\theta R_{t+h|t,t-1}\left(I+\theta(I-R_{t+h|t,t-1})\right)^{-1}(x^{i}_{t+h|t}-x^{i}_{t+h|t-1}).
\end{aligned}$$

Let me define the effective distortion parameter $\tilde{\theta}_{t,t-1}=\theta R_{t+h|t,t-1}\left(I+\theta(I-R_{t+h|t,t-1})\right)^{-1}$ reflecting the change in uncertainty $R_{t+h|t,t-1}$.

Due to information frictions, we assume that $x^{i}_{t|t}=x^{i}_{t|t-1}+K_t(s^{i}_{t}-x^{i}_{t|t-1})$ [15], and given that $\mathbb{E}^{i,\theta}_{t}(\pi_{t+h})=\begin{pmatrix} 1 & 1 \end{pmatrix}\mathbb{E}^{i,\theta}_{t}(x_{t+h})=\begin{pmatrix} 1 & \rho^{h}_{\varepsilon} \end{pmatrix}\mathbb{E}^{i,\theta}_{t}(x_t)$,

[15] K_t denotes the Kalman gain matrix.

$$
\begin{aligned}
\mathbb{E}_t^{i,\theta}(\pi_{t+h}) &= \begin{pmatrix} 1 & 1 \end{pmatrix}\left[\left(I+\tilde{\theta}_{t,t-1}\right)x^i_{t+h|t} - \tilde{\theta}_{t,t-1}x^i_{t+h|t-1}\right] \\
&= \begin{pmatrix} 1 & \rho_\varepsilon^h \end{pmatrix}\left(I+\tilde{\theta}_{t,t-1}\right)\left[x^i_{t|t-1} + K_t(s^i_t - x^i_{t|t-1})\right] - \begin{pmatrix} 1 & \rho_\varepsilon^h \end{pmatrix}\tilde{\theta}_{t,t-1}x^i_{t|t-1} \\
&= \begin{pmatrix} 1 & \rho_\varepsilon^h \end{pmatrix}x^i_{t|t-1} + \begin{pmatrix} 1 & \rho_\varepsilon^h \end{pmatrix}\left(I+\tilde{\theta}_{t,t-1}\right)K_t(s^i_t - x^i_{t|t-1})
\end{aligned}
$$

- Let us begin by considering the signal for individual i at time t which holds until the year of 2012.

$$
s^i_t = S^i_{t,\tau\varepsilon} = \begin{pmatrix} 1 & 1 \end{pmatrix}\begin{pmatrix} \tau_t \\ \varepsilon_t \end{pmatrix} + \sigma_{\nu,\tau\varepsilon}\nu^i_{t,\tau\varepsilon}
$$

  Using this, the expected inflation for individual i is given by

$$
\begin{aligned}
\mathbb{E}_t^{i,\theta}(\pi_{t+h}) &= \begin{pmatrix} 1 & \rho_\varepsilon^h \end{pmatrix}\left[x^i_{t|t-1} + \left(I+\tilde{\theta}_{t,t-1}\right)K_t(s_{i,t} - x^i_{t|t-1})\right] \\
&= \begin{pmatrix} 1 & \rho_\varepsilon^h \end{pmatrix}\left(\begin{pmatrix} \tau_{i,t|t-1} \\ \varepsilon_{i,t|t-1} \end{pmatrix} + \left(I+\tilde{\theta}_{t,t-1}\right)K_t\left(\tau_t + \varepsilon_t + \sigma_{\nu,\tau\varepsilon}\nu^i_{t,\tau\varepsilon} - \tau^i_{t|t-1} - \varepsilon^i_{t|t-1}\right)\right).
\end{aligned}
$$

  where K_t is a 2-by-1 Kalman gain matrix.

- For the year 2012 and beyond, the signal s^i_t is shifted to

$$
s^i_t = \begin{pmatrix} S^i_{t,\tau} \\ S^i_{t,\tau\varepsilon} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} \tau_t \\ \varepsilon_t \end{pmatrix} + \begin{pmatrix} \sigma_{\nu,\tau} & 0 \\ 0 & \sigma_{\nu,\tau\varepsilon} \end{pmatrix}\begin{pmatrix} \nu^i_{t,\tau} \\ \nu^i_{t,\tau\varepsilon} \end{pmatrix}.
$$

  Thus the expected inflation for individual i's updated as follows

$$
\begin{aligned}
\mathbb{E}_t^{i,\theta}(\pi_{t+h}) &= \begin{pmatrix} 1 & \rho_\varepsilon^h \end{pmatrix}\left[x^i_{t|t-1} + \left(I+\tilde{\theta}_{t,t-1}\right)K_t(s^i_t - x^i_{t|t-1})\right] \\
&= \begin{pmatrix} 1 & \rho_\varepsilon^h \end{pmatrix}\left(\begin{pmatrix} \tau^i_{t|t-1} \\ \varepsilon^i_{t|t-1} \end{pmatrix} + \left(I+\tilde{\theta}_{t,t-1}\right)K_t\left(\begin{pmatrix} S^i_{t,\tau} \\ S^i_{t,\tau\varepsilon} \end{pmatrix} - \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} \tau^i_{t|t-1} \\ \varepsilon^i_{t|t-1} \end{pmatrix}\right)\right)
\end{aligned}
$$

  where K_t is a 2-by-2 Kalman gain matrix.

Finally, the effective distortion matrix $\tilde{\theta}_{t,t-1} = \theta R_{t+h|t,t-1}\left(I+\theta(I-R_{t+h|t,t-1})\right)^{-1}$ is a 2-by-2 matrix. The first row captures how much the forecast on the trend component $\tau_{t|t}$ over-reacts to news about τ_t and ε_t. Likewise the second row implies how much expectations about $\varepsilon_{t|t}$ are distorted in response to newly received information about τ_t and ε_t.

The subjective uncertainty is

$$
\begin{aligned}
\mathbb{V}_t^{\theta}(x_{t+h}) &= \Sigma_{t+h|t}\left((1+\theta)I - \theta\Sigma_{t+h|t}\Sigma_{t+h|t-1}^{-1}\right)^{-1} \\
&= \Sigma_{t+h|t}\left(I+\theta(I-R_{t+h|t,t-1})\right)^{-1}
\end{aligned}
$$

H Proof of Proposition 4

$$
\begin{aligned}
\frac{\partial\tilde{\theta}_{t,t-1}}{\partial R_{t+h|t,t-1}} &= \frac{\partial \theta R_{t+h|t,t-1}\left(I+\theta(I-R_{t+h|t,t-1})\right)^{-1}}{\partial R_{t+h|t,t-1}} \\
&= \frac{\partial \theta R_{t+h|t,t-1}}{\partial R_{t+h|t,t-1}}\left(I+\theta(I-R_{t+h|t,t-1})\right)^{-1} + \theta R_{t+h|t,t-1}\frac{\partial\left(I+\theta(I-R_{t+h|t,t-1})\right)^{-1}}{\partial R_{t+h|t,t-1}} \\
&= \theta I\left(I+\theta(I-R_{t+h|t,t-1})\right)^{-1} - \theta R_{t+h|t,t-1}\left(I+\theta(I-R_{t+h|t,t-1})\right)^{-1}(-\theta I)\left(I+\theta(I-R_{t+h|t,t-1})\right)^{-1} \\
&= \theta\left(I+\theta(I-R_{t+h|t,t-1})\right)^{-1} + \theta R_{t+h|t,t-1}\left(I+\theta(I-R_{t+h|t,t-1})\right)^{-1}\theta\left(I+\theta(I-R_{t+h|t,t-1})\right)^{-1}
\end{aligned}
$$

For $\frac{\partial\tilde{\theta}_{t,t-1}}{\partial R_{t+h|t,t-1}} > 0$, the resulting matrix must be positive definite. Given that the identity matrix I has any non-zero vector as an eigenvector, we can assume that I and $R_{t+h|t,t-1}$ share the same set of eigenvectors. Consequently, the eigenvalues of the matrix $I+\theta(I-R_{t+h|t,t-1})$ are given by

$$1+\theta(1-\lambda_i) \text{ for } i = 1,2$$

The matrix $I+\theta(I-R_{t+h|t,t-1})$ is positive definite if and only if

$$1+\theta(1-\lambda_1) > 0 \text{ and } 1+\theta(1-\lambda_2) > 0$$

given $\theta > 0$.

Since $|\Sigma_{t+h|t}| < |\Sigma_{t+h|t-1}|$, reflecting the fact that uncertainty decreases as the information set is updated,

$$|R_{t+h|t,t-1}| = \frac{|\Sigma_{t+h|t}|}{|\Sigma_{t+h|t-1}|} < 1.$$

Because the eigenvalues of a covariance matrix represent the uncertainty within the data, the eigenvalues of the updated posterior variance are smaller compared to the eigenvalues of the prior variance. This implies that

$$\lambda_i < 1 \text{ for } i = 1,2.$$

As a result the following conditions hold.

$$1 + \theta(1 - \lambda_i) > 0 \text{ for } i = 1, 2$$

ensuring that

$$\frac{\partial \tilde{\theta}_{t,t-1}}{\partial R_{t+h|t,t-1}} > 0.$$

I Estimated Parameters in the New Keynesian Model

Parameters	Distribution	Description	mode
ϕ_π	Normal	monetary policy rule	1.3843
ϕ_x	Beta	monetary policy rule	0.0028
κ	Beta	slope of the Phillips curve	0.2122
θ	Beta	DE parameter	0.6051
ρ	Beta	persistence of a cyclical inflation	0.0258
γ	Beta	share of variance due to a cyclical inflation shock	0.8775
ρ_{is}	Beta	demand shock persistence	0.1830
ρ_{mp}	Beta	MP shock persistence	0.7386
ρ_{pc}	Beta	cost-push shock persistence	0.7331
ρ_a	Beta	tech shock persistence	0.8107
σ_u	Inv-Gamma	SD of the inflation shock	0.0154

I assume that the monetary policy shock, demand shock, and cost-push shock each follow an AR(1) process with innovations drawn from a normal distribution $\mathbb{N}(0,1)$.The sample period covers 1996Q2–2021Q4. To estimate relative uncertainty, denoted by $R_{t|t-1}$, I assume that the subjective uncertainty estimated in Figure 1 is positively related to $R_{t|t-1}$. I pin down the size of the signal noises, $\sigma_{t,\tau\varepsilon}, \sigma_{t,\tau,} \sigma_{outputsignal}$, using the standard deviation of the median SPF responses over 1996Q2–2021Q4.